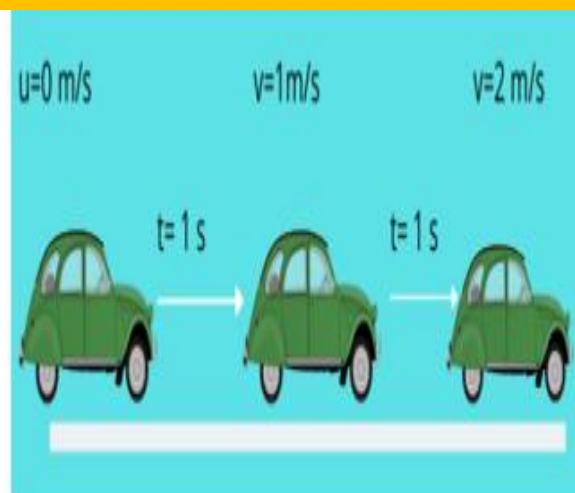
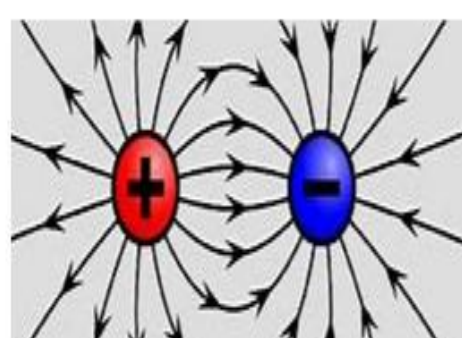




## RQF LEVEL III



**TRADE: ALL trades except (Fashion design, Fine and Plastic Arts, Agriculture, Food Processing, Animal Health, Forestry, Leather Technology, Food and Beverages Operations, Front Office and Housekeeping operations, Tourism)**



**MODULE CODE: GENAP302**

# TEACHER'S GUIDE

Module name: APPLIED PHYSICS



## APPLY GENERAL PHYSICS

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## Acronyms

<b>IC:</b>	Indicative content
<b>Av v:</b>	Average velocity
<b>ESD:</b>	Electrostatic Discharge
<b>HEP:</b>	Hydroelectric power
<b>KE:</b>	Kinetic Energy
<b>PE:</b>	Potential Energy
<b>PPE:</b>	Personal Protective Equipment
<b>R.M.S:</b>	Root Mean Square

## **Introduction**

This module describes the knowledge, skills, and attitude required to apply concepts of Physics. At the end of this module, the trainee will be able to describe basic measurements in Physics, analyze motion in one and two dimensions, demonstrate electrostatic phenomena, apply geometric optics and characterize sources of energy in the world. It will help trainee to carry out his/her specialized tasks that are useful in analyzing data, solving real life problems encountered in related fields.

## GENAP302 APPLIED PHYSICS

**Learning Outcome 1:** Describe basic measurements in Physics

**Learning Outcome 2:** Describe Motion in One Dimension

**Learning Outcome 3:** Analyse motion in Two Dimensions

**Learning Outcome 4:** Demonstrate electrostatic phenomena

**Learning Outcome 5:** Apply Geometric optics

**Learning Outcome 6:** Characterize sources of energy in the world

## LEARNING OUTCOME 1: DESCRIBE BASIC MEASUREMENTS IN PHYSICS



### Learning outcome 1. Describe basic measurements in Physics

#### Indicative contents:

- 1.1. Derivation of physical quantities
- 1.2. Calculation of experimental errors
- 1.3. Using measuring instruments at workplace.



Duration: 8 hours



## Learning outcome 1 Objectives:

By the end of the learning outcome, the trainees will be able to:

1. Describe clearly Physical quantities based on dimension analysis.
2. Calculate accurately experimental errors based on the types of errors.
3. Use properly measuring instruments based on metric system.



## Resources

Equipment	Tools	Materials
- Computer, Projector, PPT, Whiteboard and chalkboard.	- Various measuring instruments: meter stick, beam balance and stop watch, Textbooks, Scientific calculator.	- Chalks, Markers, Flipcharts.



## Advance preparation:

- Prepare in advance various measuring instruments used to measure mass, weight and time. Such as: beam balance, spring balance, stopwatch, meter ruler
- Prepare related tasks to be done by students in their groups.
- Prepare different objects to be measured like a stone, notebook
- Prepare In advances the classroom and makes proper and conducive sitting arrangements.



## 1.1: Derivation of physical quantities

A *quantity* may be defined as any observable property or process in nature with which a number may be associated. This number is obtained by the operation of measurements. The number may be obtained directly by a single measurement or indirectly, say for example, by multiplying together two numbers obtained in separate operations of measurement.

### 1.1.1 Fundamental quantities

Fundamental quantities are those quantities that are not defined in terms of other quantities. In Physics there are 7 fundamental quantities of measurements namely length, mass, time, temperature, electric current, amount of substance and luminous intensity. In this book we will study the following 3 fundamental quantities: **length, mass, time.**

#### SI units and symbols

In order to measure any quantity, a **standard unit** (base unit) of reference is chosen.

The standard unit chosen must be unchangeable, always reproducible and not subject to either the effect of aging and deterioration or possible destruction.

Before 1960, there were several systems of measurements in use around the world.

In 1960, an international system of units was established. This system is called the *International System of Units (SI)*.

The table below contains the 7 fundamental physical quantities, their SI units and symbols.

Base quantity	Name	Symbol
length	metre	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

Table 1: The seven fundamental physical quantities and SI units

### 1.1.2. Derived quantities:

Quantities which are defined in terms of the fundamental quantities via a system of quantity equations are called derived quantities.

Examples of derived quantities include area, volume, velocity, acceleration, density, weight and force.

The SI units of derived quantities are obtained from equations using mathematical expressions as follows:

**(a) Area** (e.g. for square objects) = length (m)  $\times$  length (m). The SI unit of area in symbols is  $m^2$ .

**(b) Volume** (e.g. for cubic objects) = length (m)  $\times$  length (m)  $\times$  length (m). The SI unit of volume in symbols is  $m^3$

**(c) Density** = mass (kg)/volume ( $m^3$ ). The SI unit of density in symbols is  $kg/m^3$ .

**(d) Velocity** = displacement (m)/time taken (s). The SI unit of velocity in symbols is m/s.

**(e) Acceleration** = change in velocity (m/s)/time taken (s). The SI unit of acceleration in symbols is  $m/s^2$ .

Note that some derived units have been given names. For example, force is measured in  $kg\ m/s^2$  and has been given a named unit called a newton (N). We shall encounter other derived quantities later in this course and other levels of Physics.

**Force/ Weight**=mass  $\times$  acceleration/ mass  $\times$  acceleration of gravity  $F = m \cdot a$ ,  $W = m \cdot g$

1 N (newton) =  $1kg \cdot m/s^2$

### 1.1.3. Prefixes for SI units

Physical quantities are of wide range of magnitude. For example, the mass of earth is about  $6 \times 10^{24}kg$  while the diameter of a molecule is about  $10^{-10}m$ . Writing such quantities is not an easy task for everyone. Some words have been used with SI units as short-cut to writing such magnitude. These words are associated with certain magnitude. For example, the prefix like milli stands for  $10^{-3}$ , while the prefix kilo stands for  $10^3$ . Since these words are used or fixed before the SI units, they are called prefixes.

Prefixes	Value	Standard form	Symbol
Tera	1 000 000 000 000	$10^{12}$	T
Giga	1 000 000 000	$10^9$	G
Mega	1 000 000	$10^6$	M
Kilo	1 000	$10^3$	k
deci	0.1	$10^{-1}$	d
centi	0.01	$10^{-2}$	c
milli	0.001	$10^{-3}$	m
micro	0.000 001	$10^{-6}$	$\mu$
nano	0.000 000 001	$10^{-9}$	n
pico	0.000 000 000 001	$10^{-12}$	p

Table 2: Prefix, symbol and magnitude of SI units

**Example 1:** Raju and his friend Akhil live 2000 m from each other. Express the distance between their houses in kilometres (km).

**Solution:** We know that 1000 m = 1 km

therefore, 2000 m = 2 km

therefore, the distance between the two houses is 2 km.

#### 1.1.4. Dimension

In dimensional analysis, dimensions express physical quantities in terms of fundamental dimensions such as mass, length, and time.

In Physics, the dimensions of a physical quantity are the product of its mass, length, and time units. For example, the dimensions of velocity are mass (M) length (L) per time (T).

**Example:** Length may have units of meters, centimetres, hectometres, millimetres or micrometers; but any length always has a dimension of L, independent of what units are arbitrarily chosen to measure it.

The physical quantity, speed, may be measured in units of metres per second; but regardless of the units used, speed is always a length divided by a time, so we say that the dimensions of speed are length divided by time, or simply l/t.

Similarly, the dimensions of area are L<sup>2</sup> since area can always be calculated as a length times a length. Two different units of the same physical quantity have conversion factors that relate them.

	Quantity	SI unit	Abb.	Dimension
1	Length	meter	m	[L]
2	Mass	kilogram	kg	[M]
3	Time	second	s	[T]
4	Electric current	ampere	A	[A]
5	Temperature	kelvin	K	[K]
6	Luminous intensity	candela	cd	[cd]
7	Amount of substance	mole	mol	[mol]

Table 3: Base quantities and dimensions used in the SI

### Dimensional analysis

The fact that an equation must be homogenous enables predictions to be made about the way in which physical quantities are related to each other. Examples of the method are given in the table below:

	Quantity	Definition	Formula	Units	Dimensions
Basic Mechanical	Length	Fundamental	d	m (meter)	L (Length)
	Time	Fundamental	t	s (second)	T (Time)
	Mass	Fundamental	m	kg (kilogram)	M (Mass)
	Area	length <sup>2</sup>	$A = d^2$	m <sup>2</sup>	L <sup>2</sup>
	Volume	length <sup>3</sup>	$V = d^3$	m <sup>3</sup>	L <sup>3</sup>
	Density	$\frac{\text{mass}}{\text{volume}}$	$\rho = \frac{m}{V}$	kg/m <sup>3</sup>	$\frac{M}{L^3}$
	Velocity	$\frac{\text{length}}{\text{time}}$	$v = \frac{d}{t}$	m/s c (speed of light)	$\frac{L}{T}$
	Acceleration	$\frac{\text{velocity}}{\text{time}}$	$a = \frac{v}{t}$	m/s <sup>2</sup>	$\frac{L}{T^2}$
	Momentum	mass × velocity	$p = m \cdot v$	kg·m/s	$\frac{ML}{T}$

Basic Mechanical	Force Weight	mass × acceleration mass × acceleration of gravity	$F = m \cdot a$ $W = m \cdot g$	N (newton) = $\text{kg} \cdot \text{m}/\text{s}^2$	$\frac{ML}{T^2}$
	Pressure	$\frac{\text{force}}{\text{area}}$	$p = \frac{F}{A}$	Pa (pascal) = $\text{N}/\text{m}^2 = \text{kg}/(\text{m} \cdot \text{s}^2)$	$\frac{M}{LT^2}$
	Energy or Work Kinetic Energy Potential Energy	$\frac{\text{force} \times \text{distance}}{2}$ $\frac{\text{mass} \times \text{velocity}^2}{2}$ mass × acceleration of gravity × height	$E = F \cdot d$ $K = \frac{1}{2} mv^2$ $U = m \cdot g \cdot h$	J (joule) = $\text{N} \cdot \text{m} = \text{kg} \cdot \text{m}^2/\text{s}^2$	$\frac{ML^2}{T^2}$
	Power	$\frac{\text{energy}}{\text{time}}$	$P = \frac{E}{t}$	W (watt) = $\text{J}/\text{s} = \text{kg} \cdot \text{m}^2/\text{s}^3$	$\frac{ML^2}{T^3}$
Thermal	Temperature	Fundamental	K	°C (celsius), K (kelvin)	K (Temp.)
	Heat	heat energy	$Q = mc\Delta t$	J (joule) = $\text{kg} \cdot \text{m}^2/\text{s}^2$	$\frac{ML^2}{T^2}$

Table 4: Dimensions of Physical quantities



### Theoretical learning Activity

In groups of four, brainstorm and try to find answers to the following questions:

1. Define the term “ quantity”
2. What is the difference between fundamental and derived quantity?
3. Give out four examples of fundamental and four derived quantities
4. Define volume and state its SI unit



### Practical learning Activity

In pairs, ask learners to choose any object and measure its length and express their answer in different units (e.g., meters, centimetres).



## Points to Remember

*Fundamental quantities* are those quantities that are not defined in terms of other quantities.

***Derived quantities:*** Quantities which are defined in terms of the fundamental quantities via a system of quantity equations

***In dimensional analysis,*** we express physical quantities in terms of fundamental dimensions such as mass, length, and time.



## 1.2: Calculation of experimental errors

A **measurement** is an observation that has a numerical value and often a unit. When you measure an object, you compare it with a standard unit. Every measurement must be expressed by a number and a unit.

### 1.2.1. Types of errors

The experimental error can be defined as: “the difference between the observed value and the true value” (Merriam-Webster Dictionary). The uncertainties in the measurement of a physical quantity (errors) in experimental science can be separated into two categories: random and systematic.

1. **Random errors** fluctuate from one measurement to another. They may be due to: poor instrument sensitivity, random noise, random external disturbances, and statistical fluctuations (due to data sampling or counting). A random error arises in any measurement, usually when the observer has to estimate the last figure possibly with an instrument that lacks sensitivity. Random errors are small for a good experimenter and taking the mean of a number of separate measurements reduces them in all cases.
2. **Systematic errors**: is an error created by a mistake in the way the experimental procedure is carried out and can be caused by the instruments or equipment being used, a change in the environment, or errors in how the experiment is carried out.

Systematic errors can be at least minimized by instrument calibration and appropriate use of equipment.

A systematic error may be due to an incorrectly calibrated instrument, for example a ruler or an ammeter. Repeating the measurement does not reduce or eliminate the error and the existence of the error may not be detected until the final result is calculated and checked, say by a different experimental method. If the systematic error is small a measurement is **accurate**.

There are two main causes of error: **human** and **instrument**.

- A. **Human error** can be due to mistakes (misreading 22.5cm as 23.0cm) or random differences (the same person getting slightly different readings of the same measurement on different occasions).
- B. **Instrument errors** occur when the reading on an instrument is different from the true value being measured. This can be caused by the instrument being calibrated incorrectly. For example, if the scales in the image below read 6g when there is nothing on them, then this will introduce an error of 6g into any readings made with them. In this case, the true mass of the strawberries would be 140g.



Figure 1: Some strawberries being weighed on a digital scale.

It is usually easy to correct by recalibrating the instrument and readings.

### 1.2.2.1. Calculations of errors

When combining measurements in a calculation, the uncertainty in the final result is larger than the uncertainty in the individual measurements. This is called **propagation of uncertainty** and is one of the challenges of experimental Physics.

There are simple rules that can provide a reasonable estimate of the uncertainty in a calculated result:

#### 1. Absolute and Relative Errors (Uncertainties)

**A. Absolute error** is an expression of how far a measurement is from its actual or expected value.

Or is a measure of how much an observed value differs from the true value or an expected value.

**Absolute Error (AE) = |Observed Value–True Value|**

**For example**, if you measure the length of an object and the true length is 10 cm, but your measurement is 9.5 cm, the absolute error would be  $|10\text{ cm} - 9.5\text{ cm}| = 0.5\text{ cm}$

This means your measurement deviates from the true value by **0.5 cm**.

**B. Relative error** (sometimes called proportional error) is expressed as the ratio of the absolute error to the magnitude of the true value. It is often represented as a percentage. The formula for relative error (RE) is given by:

$$\text{Relative Error (RE)} = \left| \frac{\text{Observed Value} - \text{True Value}}{\text{True Value}} \right|$$

In mathematical terms, it can be written as:

**For example**, if you measure the length of an object and the true length is 10 cm, but your measurement is 9.5 cm, the absolute error is  $|10 \text{ cm} - 9.5 \text{ cm}| = 0.5 \text{ cm}$

The relative error would be:

$$RE = \frac{\text{ABSOLUTE ERROR}}{\text{TRUE VALUE}} = \frac{0.5 \text{ cm}}{10 \text{ cm}} = 0.05$$

Note that the relative error has no unit.

**C. Percentage error**

When the relative error is expressed as a percentage, it is called a **percentage error**.

For the example above,

$$RE = \left| \frac{9.5 \text{ cm} - 10 \text{ cm}}{10 \text{ cm}} \right| \times 100\% = 5\%$$

### 1.2.2. Significant figures of measurements

Significant figure represents the number of meaningful digits in a measurement.

#### 1.2.2.1. The rules for identifying significant digits

The **rules** for deciding which digits in a measurement are significant are as follows:

**Rule 1:** All nonzero digits in a measurement are significant.

- 237 has three significant figures.
- 1.897 has four significant figures.

**Rule 2:** Zeros that appear between other nonzero digits (i.e., "**middle zeros**") are always significant.

- 39,004 have five significant figures.
- 5.02 have three significant figures.

**Rule 3:** Zeros that appear in front of all of the nonzero digits are called **leading zeros**.

Leading zeros are never significant.

- 0.008 has one significant figure.
- 0.000416 has three significant figures.

**Rule 4:** Zeros that appear after all nonzero digits are called **trailing zeros**. A number with trailing zeros that lacks a decimal point may or may not be significant.

- $1.4 \times 10^3$  has two significant figures.
- $1.40 \times 10^3$  has three significant figures.
- $1.400 \times 10^3$  has four significant figures.

**Rule 5:** Trailing zeros in a number with a decimal point are significant. This is true whether the zeros occur before or after the decimal point.

- 620.0 has four significant figures.
- 19.000 has five significant figures.

### Example

Give the number of significant figures in each. Identify the rule for each.

a. 5.87

c. 52.90

e. 500

b. 0.031

d. 00.2001

### Solution:

a. 5.87, three significant figures

d. 00.2001, four significant figures

b. 0.031, two significant figures

e. 500, 1 significant figure

c. 52.90, four significant figures

### 1.2.2.2. Rounding off numbers

Rounding off numbers is the process of approximating a numerical value to a specified degree of precision. The rules for rounding off numbers depend on the desired precision and the digit immediately following the digit to be rounded.

### Rules applied when rounding off whole numbers

- ✓ If the first of the digits to be dropped (reading from left to right) is 1, 2, 3 or 4, simply replace all dropped digits with the appropriate number of zeros. For example, 57,384 rounded off to the nearest thousands becomes 57,000.
- ✓ If the first of the digits to be dropped (reading from left to right) is 6, 7, 8 or 9, increase the preceding digit by 1. For e.g., 5,383 rounded off to the nearest hundred becomes 5,400.
- ✓ If only one digit is to be dropped and this digit is 5, increase the preceding digit by 1 if it is odd, and leave it unchanged if it is even. Thus, if 685 is to be rounded off to the nearest tens it becomes 680, while 635 rounded off to the nearest tens becomes 640.

### Rules applied when rounding off decimal numbers:

- ✓ Rounding 3.678 to two decimal places:

The digit in the third decimal place is 8 (greater than 5), so we round up.

The result is 3.68.

- ✓ Rounding 2.314 to one decimal place:

The digit in the second decimal place is 1 (less than 5), so we leave the digit in the rounding place unchanged.

The result is 2.3.

- ✓ Rounding 876.94 to three significant figures:

The digit in the fourth place is 9 (greater than 5), so we round up.

The result is 877.



### Theoretical learning Activity

In group of four, discuss on the following activity

Round off to;

- a) The nearest unit: 6.8; 10.5; 801.625,
- b) The nearest tenth 5.83; 480.625; 0.234; 0.285; 6.58; 36.092,
- c) The nearest hundredth: 3.632; 812.097; 0.71

d) The nearest thousandth: 0.2827; 0.0066.

e) The nearest tens: 56; 44; 17; 656,

f) The nearest hundreds: 219; 256; 71,550; 930.7,

g) The nearest thousands: 890; 1600; 10 500; 13 856; 5420.5



### **Practical learning Activity**

Trainees perform measurement activities:

#### **Materials:**

1. Measurement tools (ruler, digital calliper)
2. Whiteboard or chart paper
3. Markers
4. Examples of measurements with various significant figures
5. Calculator

#### **Measurement Exercise:**

- a. Provide students with various objects and tools for measurements.
- b. In groups, students measure different quantities and determine the number of significant figures using the rules.
- c. Record measurements and significant figures on the whiteboard or chart paper.

#### **Group Discussion:**

Discuss the results as a class, emphasizing how the rules for significant figures were applied.

Identify any challenges or questions that arise.



## Points to Remember

**A measurement** is an observation that has a numerical value and unit

**Error:** “the difference between the observed value and the true value”

**Random errors** fluctuate from one measurement to another. They may be due to: poor instrument sensitivity, random noise, random external disturbances, and statistical fluctuations (due to data sampling or counting).

**A systematic error** may be due to an incorrectly calibrated instrument, for example a ruler or an ammeter.

There are two main causes of error: **human** and **instrument**.

**Accuracy** is the degree of veracity (“how close to true”) while **precision** is the degree of reproducibility (“how close to exact”).

Measurements involve comparing an unknown quantity with a known fixed unit quantity (standard unit).

### 1.3.1. Measurement of length

Length is measured in metres (m).

Length is measured in metres. One metre is the distance between the two marks on a standard platinum-iridium bar kept at Paris (France). Although the metre is the standard unit of length, it is sometimes too big to measure some distances and too small to measure others. We therefore need other larger and smaller units related to the metre to carry out some measurements.

Table 1.5 shows the SI units of length and its relationship with other larger and smaller units of length.

## UNITS OF LENGTH CONVERSION CHARTS

1 kilometre (km)	= 10 Hectometres (hm)	= 1000 m
1 Hectometre (hm)	= 10 Decametres (dcm)	= 100 m
1 Decametre (dcm)	= 10 Metres (m)	
1 Metre (m)	= 10 Decimetres (dm)	= 100 cm = 1000 mm
1 Decimetre (dm)	= 10 Centimetres (cm)	
1 decimeter	= 0.1 meter	
1 Centimetre (cm)	= 10 Millimetres (mm)	
1 centimeter	= 0.01 meter	
1 millimeter	= 0.001 meter	

*Table 5: Units of length and their symbols.*

Let us now discuss some of the instruments used to measure length.

**Meter rule:** Straight distances which are less than one metre in length are generally measured using metre rules. Metre rules are graduated in millimetres (mm). Each division on the scale represents 1 mm unit.



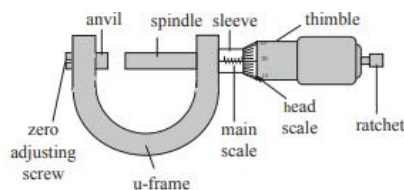
*Figure 2: A metre rule*



**Tape measure:** A tape measure is a flexible instrument used for measuring lengths and distances; an essential for many industries and applications.

Figure 3: Tape measure

### Micrometer Screw gauge



*Figure 4: Photograph of a micrometer screw gauge*

## Vernier calliper

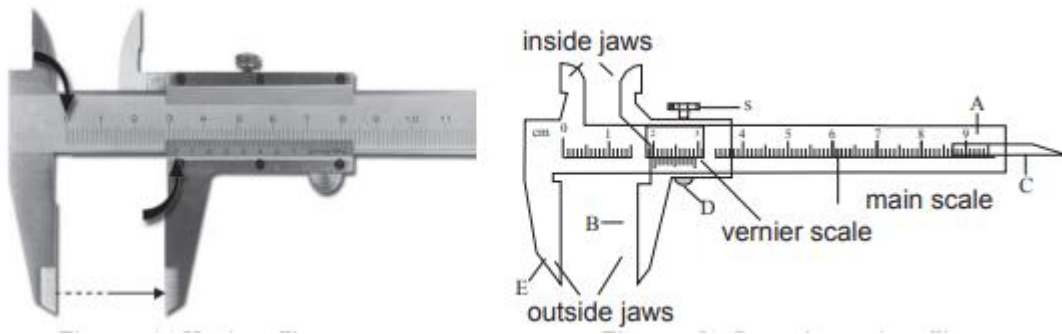


Figure 5: Photograph of a Vernier calliper

### 1.3.2. Measurement of mass

Mass is the amount of matter in a substance. Its SI unit is kilogram (kg).



The measuring instruments used to measure mass are Beam balance, Electronic balance and kitchen balance.

Figure 6: Kitchen balance



Figure 7: Beam balance



Figure 8: Electronic balance

### 1.3.3. Measurement of time

Time is a measure of the duration taken by an event. It is measured with a Stop watch



Figure 9: Analog stop watch



Figure 10: Digital stopwatch

All living things have an inbuilt biological clock which seems to control the rhythm of their life cycle. For example, the cock will crow only at specific time intervals.



#### Theoretical learning Activity

Individually, learners match the instrument with its corresponding quantity measurement.

Instrument		Quantity measurement
Meter rule		Time
Thermometer		Small volume
Beam balance		Lengths in mm, cm, m
Stop watch		Temperature
Vernier calliper		Mass
Micrometre screw gauge		Diameters of objects
Burette		Very small lengths such as the thickness of hair

Table 6: Instruments and quantity of measurement



### **Practical learning Activity:**

In group of four learners, measure the length and width of any rectangular shape and record down measurements.



### **Points to Remember**

#### **The instruments used to measure length.**

1. Meter stick
2. Meter ruler
3. Decametre
4. Screw gauge
5. Vernier calliper

#### **The instruments used to measure mass are:**

1. Beam balance
2. Electronic balance
3. Spring balance

#### **The instruments used to measure time.**



## Learning outcome 1 Formative Assessment

### Written Assessment

#### 1. Multiple choice

A. The number of significant digits in 0.0006032 is a) 8; b) 7; **c) 4**; d) 2

B. The length of a body is measured as 3.51m. If the accuracy is 0.01 m, then the percentage error in the measurement is;

- a) 351 %;      b) 1 %;  
**c) 0.28 %;**      d) 0.035 %

C. The dimensional formula for gravitational constant is;

- a)  $M^1L^3T^{-2}$                       **b)  $M^{-1}L^3T^{-2}$**   
c)  $M^{-1}L^{-3}T^{-2}$                       d)  $M^1L^{-3}T^2$

D. The velocity of a body is expressed as  $v = (x/t) + yt$ .

The dimensional formula for x is;

- a)  $ML^0T^0$                       b)  $M^0LT^0$   
c)  $M^0L^0T$                       **d)  $MLT^0$**

#### 2. Name three fundamental quantities and their SI units.

##### Solution

- Length, SI Unit: Meter (m)
- Mass, SI Unit: Kilogram (kg)
- Time, SI Unit: Second (s)

3. Distinguish between a fundamental (base) quantity and a derived quantity. Give one example of each.

##### Solution

Fundamental (Base) Quantity: A fundamental or base quantity is a physical quantity that is chosen by convention and is independent of other physical quantities. Example: length, mass, and time.

Derived Quantity: it is a quantity that can be expressed in terms of fundamental quantities. The units for derived quantities are derived from the base units. Velocity is an example of a derived quantity

4. Give a reason why it was necessary to establish SI units.

**Solution**

The establishment of SI units was necessary for global standardization, ensuring consistent and precise communication in science, technology, and trade. It promotes accuracy, interdisciplinary compatibility, and ease of use, allowing for universal understanding and collaboration.

5. How many micrometres are there in 4 cm?

**Solution**

One centimeter is equal to 10,000 micrometers. Therefore, to find out how many micrometers are in 4 centimeters?

$$4 \text{ cm} \times 10,000 \text{ micrometers/cm} = 40,000 \text{ micrometers}$$

So, there are 40,000 micrometers in 4 centimeters.

6. Express the following in millimetres: (a) 2.7 m (b) 26.9 cm (c) 356  $\mu\text{m}$ .

**Solution**

To express the given measurements in millimetres:

(a) 2.7m is equivalent to 2,700mm because  $1\text{m}=1,000\text{mm}$ .

(b) 26.9cm is equivalent to 269mm because  $1\text{cm}=10\text{mm}$ .

(c) To convert micrometers ( $\mu\text{m}$ ) to millimetres (mm), you need to divide the value in micrometers by 1000, as there are 1000 micrometers in 1 millimetre.

$$356\mu\text{m} = 356/1000\text{mm} = 0.356\text{mm}$$

So, 356 $\mu\text{m}$  is equivalent to 0.356 mm

Therefore:

(a)  $2.7\text{m}=2,700\text{mm}$

(b)  $26.9\text{cm}=269\text{mm}$

(c)  $356\mu\text{m}=0.356\mu\text{m}$

7. Round off the following measurement so that all have the same degree of accuracy:  
468.5m; 0.00708m; 3.467m; 56.93m; 3.004m

### Solution

To round off the measurements to the same degree of accuracy, we need to choose a common decimal place. Let's choose the hundredth place (two decimal places) for consistency:

- 468.5 m:

Already rounded to the desired accuracy (two decimal places).

468.5 m

- 0.00708 m:

Rounded to two decimal places: 0.01 m

- 3.467 m:

Rounded to two decimal places: 3.47 m

- 56.93 m:

Rounded to two decimal places: 56.93m

- 3.004 m:

Rounded to two decimal places: 3.00 m

8. Round off the numbers below to the shown number of significant figures in the brackets:

a) 245 086 (4) →**245,100**

c) 8 465 (3) →**8,470**

b) 406.50 (3) →**407**

d) 84.25 (2) →**84**

## Practical assessment

### Title: Investigating types of errors

#### Materials:

1. Tape measure
2. Table

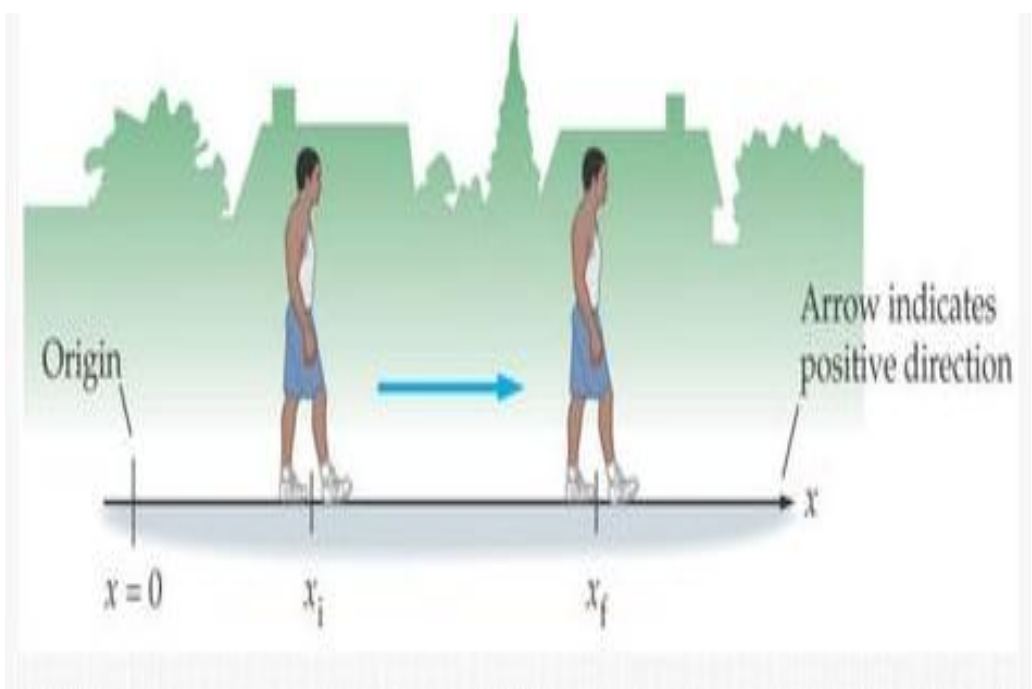
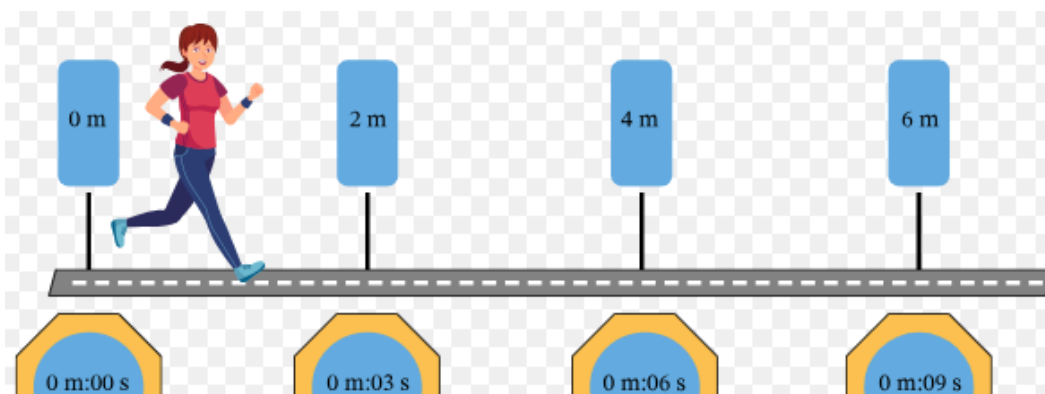
#### Procedure:

- 1) Using the tape-measure, measure the length of your table and record the result.
- 2) Repeat the same measurement several times and record the results.
- 3) Compare your findings.

#### Questions:

1. Are your results the same?
2. (If not) what may have caused the differences?
3. Where do you think errors come from?

## Learning Outcome 2: Describe Motion in 1-Dimension



### Learning outcome 2. Describe Motion in 1- Dimension

#### Indicative contents

- 2.1. Explanation of displacement, velocity and acceleration concepts
- 2.2. Illustration of linear motion using corresponding graphs
- 2.3. Application of equations of motion



**Duration: 5 hours**



### **Learning outcome 2 Objectives**

**By the end of the learning outcome, the trainees will be able to:**

1. Explain clearly displacement, velocity and acceleration concepts in 1 dimension
2. Illustrate clearly Linear motion concepts using corresponding graphs
3. Apply appropriately equations of motion in a straight line under constant acceleration



### **Resources**

Equipment	Tools	Materials
- PPE, whiteboard, chalkboard, computer, projector, textbooks	- Scientific calculator, meter ruler, compass	- Chalks, markers



### **Advance preparation:**

- Prepare a Simulation video of a different objects moving along a straight path.
- Prepare relevant pictures highlighting distance and displacement.



## 2.1: Explanation of displacement, velocity and Acceleration

### concepts

#### 2.1.1. Displacement and distance

In one-dimensional geometry, locating a point on a line involves specifying its position along the line.

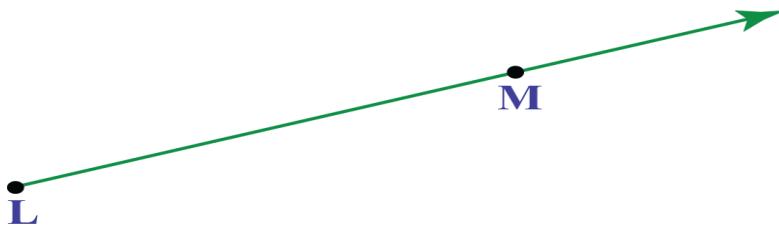


Figure 11: Point on a line

#### Distance

Distance is the total length of the path followed by an object, regardless of the direction of motion. It is a scalar quantity and measured in units of length. The SI unit of distance is the metre (m). Long distances may be measured in kilometres (km) while short distances may be measured in centimetres (cm) or millimetres (mm). It should be noted that in determining the distance between two points, the direction at any point along the path is not considered.

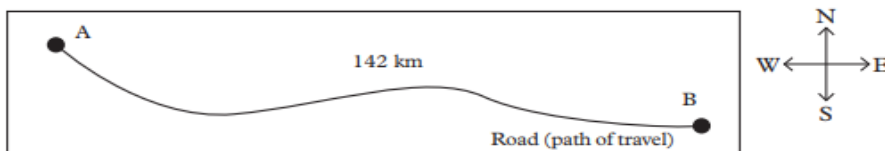


Figure 12: Distance between points A and B (Body taking different paths)

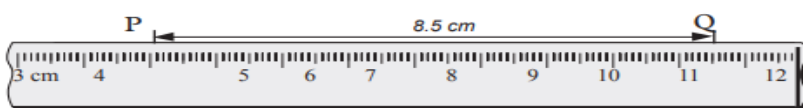
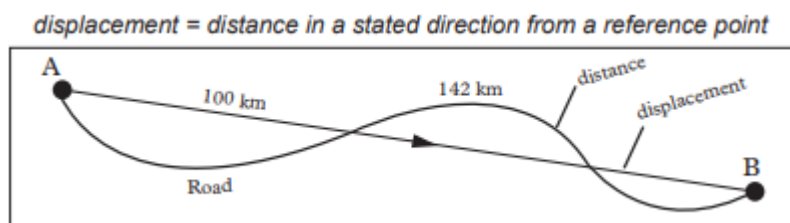


Figure 13: Distance between points P and Q (Body following the same path)

## Displacement

Displacement is the object's overall change in position from the starting to the end point. It is the shortest distance along a straight line between two points in the direction of motion.

The SI unit of displacement is the metre (m).



## Speed

The distance moved by a body per unit time is called **speed**. In this motion, direction is not considered. Thus, Speed = distance moved/time taken:

$$v = \frac{d}{t}$$

Where v is the speed in m/s, d is the distance in metres and t is the time taken in seconds

The SI unit of speed is metres per second (m/s). Other units of speed such as kilometres per hour (km/h) and centimetres per second (cm/s) are also in common use.

When a body covers equal distances in equal time intervals, it is said to move with uniform speed.

**Example 2.1** What is the speed of a racing car in metres per second if the car covers 360 km in 2 hours?

### Solution

Speed = distance moved/time taken

$$360 \text{ km}/2 \text{ h} = 180 \text{ km/h}$$

$$\text{OR, Speed} = \text{distance/moved time taken} = 360 \times 1000 \text{ m}/2 \times 3600 = 50 \text{ m/s}$$

## Velocity

The speed of a body in a specified direction is called velocity or velocity is the rate of change of distance in a particular direction.

Therefore, Velocity = distance moved in a particular direction/time taken

$$v = \frac{s}{t}$$

Where v is the velocity in m/s, s is the displacement in metres and t is the time taken in seconds.

Velocity is also defined as the displacement covered per unit time or the rate of change of displacement. i.e. Velocity = displacement/time taken

The SI unit of velocity is metres per second (m/s).

In some cases, the velocity of a moving body keeps on changing. In such cases, its average velocity of the body is considered.

Average velocity = total displacement/time taken

### Example

A car travelled from town A to town B 200 km east of A in 3 hours. It then changed direction and travelled a distance of 150 km due north from town B to town C in 2 hours. (Fig. 2.9).

Calculate the average

- speed for the whole journey.
- velocity for the whole journey.

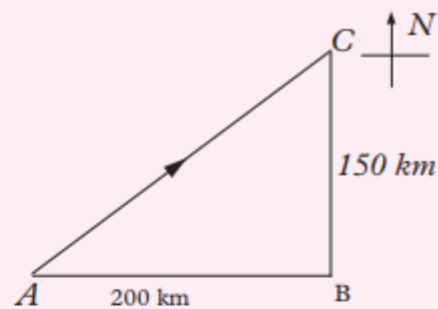


Fig. 2.9: Town A, B and C

### Solution

$$(a) \text{ Average speed} = \frac{\text{total distance}}{\text{time taken}} = \frac{(200 + 150) \text{ km}}{(3 + 2) \text{ h}}$$

$$= \frac{350}{5} \left( \frac{\text{km}}{\text{h}} \right)$$

$$= 70 \text{ km/h}$$

$$(b) \text{ Average velocity} = \frac{\text{displacement, AC}}{\text{time taken}} = \frac{\sqrt{200^2 + 150^2} \text{ km}}{3 + 2 \text{ h}}$$

$$= \frac{250}{5} \text{ km}$$

$$= 50 \text{ km/h, Direction is from A to C}$$

$$\text{In m/s} = \frac{50\,000 \text{ m}}{3\,600 \text{ s}}$$

$$= 13.89 \text{ m/s}$$

### 2.1.2. Description of average and instantaneous acceleration

Acceleration is defined as the rate of change of velocity i.e. Acceleration = Change in velocity/Time taken

$$a = \frac{v}{t}$$

Where a is acceleration in metres per square seconds, v is the velocity in metres per second and t is the time taken in seconds

The SI unit of acceleration is metres per square second or m/s<sup>2</sup>

**Note** that if the acceleration of a body is  $4 \text{ m/s}^2$ , it means that its velocity is increasing by  $4 \text{ m/s}$  every second. When the velocity of a body decreases, it is said to be **decelerating** or **retarding**. Deceleration or retardation refers to a negative acceleration. This is usually shown with a negative sign before the value. Forexample,  $a = -4 \text{ m/s}^2$  shows a deceleration at  $4\text{m/s}^2$ .

A body moving with uniform velocity has zero acceleration since there is no change in velocity. When the rate of change of velocity with time is constant, the acceleration is referred to as uniform acceleration.

**Example 1.** If an object gains a velocity of  $10\text{m/s}$  in  $5\text{s}$ , its average acceleration is  $a = \text{change in velocity}/\text{time taken} = 10\text{m/s}/5\text{s} = 2\text{ms}^{-2}$

**Example 2.** A motor car is uniformly retarded and brought to rest from a speed of  $108 \text{ km/h}$  in  $15 \text{ s}$ . Find its acceleration.

Answer: Given:  $u = 108 \text{ km/h} = 30 \text{ m/s}$  and  $v = 0 \text{ m/s}$

$$a = \Delta v/\Delta t = v-u/\Delta t = 0 - 30/15 = -2\text{m/s}^2$$

The minus sign here simply means that the car is accelerating in the opposite direction to its initial velocity.

Average acceleration is a measure of how quickly the velocity of an object changes over a given time interval. It is calculated as the change in velocity divided by the change in time.

The formula for average acceleration ( $a_{\text{avg}}$ ) is:

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t}$$

Where:

$\Delta v$  is the change in velocity (final velocity ( $v_f$ ) minus initial velocity ( $v_i$ )),

$\Delta t$  is the change in time (final time ( $t_f$ ) minus initial time ( $t_i$ )).

$$a_{\text{avg}} = \frac{v_f - v_i}{t_f - t_i}$$

### Example

A car accelerates from rest to a final speed of  $30 \text{ m/s}$  in a time interval of  $10 \text{ seconds}$ .

**Given:**

Initial velocity ( $v_i$ ):  $0 \text{ m/s}$  (starting from rest)

Final velocity ( $v_f$ ):  $30 \text{ m/s}$

Initial time ( $t_i$ ):  $0 \text{ seconds}$

Final time ( $t_f$ ): 10 seconds

**Solution:**

$$a_{\text{avg}} = \frac{v_f - v_i}{t_f - t_i}$$

$$a_{\text{avg}} = \frac{30 \text{ m/s} - 0}{10 \text{ s} - 0} = \frac{30 \text{ m/s}}{10 \text{ s}} = 3 \text{ m/s}^2$$

### 2.1.2. Description of average and instantaneous acceleration

The instantaneous acceleration of a body is the acceleration the body has at a particular time, at a specific point of its trajectory. To define the concept of instantaneous acceleration with precision we must begin with the average acceleration in an interval and make it infinitely small. This process is similar to the one we followed to calculate instantaneous velocity from average velocity.

The **instantaneous acceleration**, or simply acceleration, is defined as the limit of the average acceleration when the interval of time considered approaches 0. It is also defined in a similar manner as the derivative of velocity with respect to time.

**Solved example:** A car accelerates from rest to a velocity of 20 m/s in 5 s. Thereafter, it decelerates to a rest in 8 s. Calculate the acceleration of the car (a) in the first 5 s, (b) in the next 8 s.

**Solution**

(a) Acceleration = $\frac{\text{change in velocity}}{\text{time taken}}$	(b) Acceleration
$= \frac{\text{final velocity} - \text{initial velocity}}{\text{time taken}}$	$\frac{\text{final velocity} - \text{initial velocity}}{\text{time taken}}$
(rest means velocity is zero)	$= \frac{(0 - 20) \text{ m/s}}{8 \text{ s}}$
$= \frac{(20 - 0 \text{ m/s})}{5 \text{ s}}$	$= \frac{-20 \text{ m/s}}{8 \text{ s}} = -2.5 \text{ m/s}^2$
$= 4 \text{ m/s}^2$	or deceleration of $2.5 \text{ m/s}^2$



## Theoretical learning Activity

In groups of four, discuss on the following problems

1. A car starts from rest and is accelerated uniformly at the rate of  $2\text{m/s}^2$  for 6s. It then maintains a constant speed for half a minute. The brakes are then applied and the vehicle uniformly retarded to rest in 5s. Find the maximum speed reached in km/h and the total distance covered in meters.
2. What is the velocity of an object, at rest, if it experiences a constant acceleration of  $10\text{m/s}^2$  to the right after a period of 3s?
3. As a bus comes to stop, it slows from  $9.00\text{m/s}$  to  $0.00\text{m/s}$  in 360s. Find the average acceleration of the bus.
4. Consider a ball thrown upward with an initial velocity of  $20\text{ m/s}$ . What will its velocity be after 3s if it undergoes a constant acceleration of  $a = 10\text{m/s}^2$  downward?

$$a = -10\text{m/s}^2$$

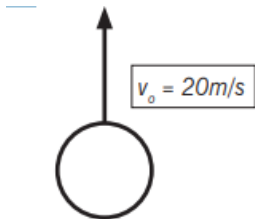


Figure 14: Ball thrown vertically upward with an initial velocity of  $20\text{m/s}$

5. Distinguish between: (a) Speed and velocity. (b) Distance and displacement.
6. Rusangwanwa cycles to school 2.5 km away in 5 minutes. What is his average speed in (a) metres per second? (b) Kilometres per hour?
7. Nesa and Nshimiye decided to walk to a picnic site 12 km away. They walked the first 6 km at an average speed of 6 km/h and the rest at 5 km/h. (a) How long did the journey take? (b) What was their average speed for the journey?



## Practical learning Activity

### Materials:

- 1) Compass or directional indicators
- 2) Stopwatch or timer
- 3) Meter ruler
- 4) Chart paper and markers

### Procedure:

### Introduction:

Introduce the concept of velocity as speed in a specified direction.

### Activity:

- 1) Assign each group a specific direction (north, south, east, and west).
- 2) Ask each group to measure and record the time taken for a person to walk or run a specified distance in that direction.
- 3) Calculate the velocity using the formula  $\text{Velocity} = \frac{\text{Displacement}}{\text{Time}}$
- 4) Discuss and compare the results on the chart paper.

### Discussion:

Explore the difference between speed and velocity and discuss scenarios where velocity is important.



## Points to Remember

**Distance** is the total length of the path followed by an object, regardless of the direction of motion.

**Displacement** is the object's overall change in position from the starting to the end point.

$$\text{Speed} = \frac{\text{distance moved}}{\text{time taken}}$$

$$\text{Velocity} = \frac{\text{distance moved in a particular direction}}{\text{time taken}}$$

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time taken}}$$



## 2.2 : Illustration of linear motion using corresponding graphs

### 2.2.1. Slopes and General Relationships

**Definition of rise:** The vertical change between two points is called the rise.

**Definition of run:** The horizontal change between any two points is called the run

**Definition of slope:** The slope equals the rise divided by the run. That is, Slope = rise/run

We usually determine the slope of a line from its graph by looking at the rise and run.

$$\text{Slope} = \frac{\text{Vertical Change}}{\text{Horizontal Change}} = \frac{\text{Rise}}{\text{Run}}$$

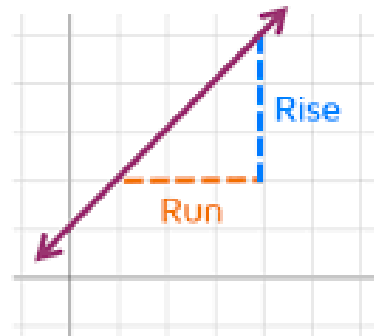


Figure 15: Rise, Run and slope of a line

### Definition of intercept points

Obstruct (someone or something) so as to prevent them from continuing to a destination.

### Define equation of a straight line

$$Ax + By + C = 0$$

A straight line is defined by a linear equation whose general form is  $AX + bY + C = 0$ , where A, B are not both 0. The coefficients A and B in the general equation are the components of vector  $n = (A, B)$  normal to the line.

## 2.2.2. Graph of Displacement vs. Time

### Constant velocity

This general graph represents the motion of a body travelling at constant velocity. The graph is linear (that is, a straight line). Recall that linear equations have the general form.  $y = mx$  (where  $m$  is a constant and  $x$  is a variable). The number  $m$  is called the slope of the line (the vertical rise over the horizontal run).

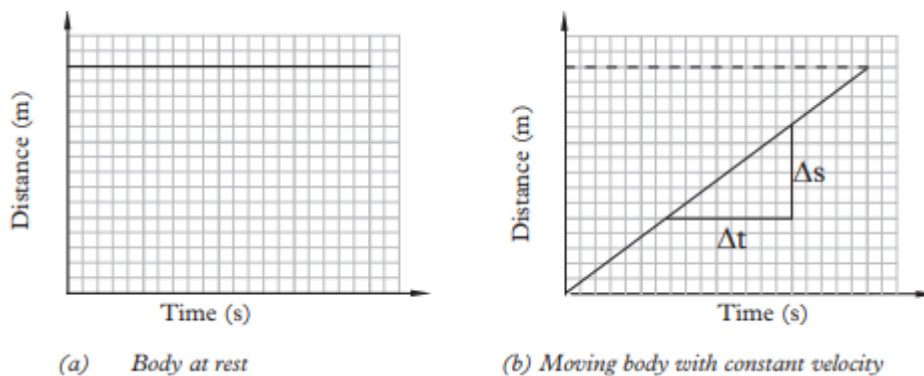


Figure 16: (a) and (b): Distance-time graph for a body at rest and moving with a constant speed

The graph in figure 16(a) shows that the distance covered by the body is not changing with time. The body is therefore at rest (stationary). The graph in figure 16(b) shows that the distance covered by the body is increasing with time. The gradient of the graph is  $\Delta s/\Delta t$  and represents the speed of the object. Thus, the graph represents the motion of the body moving with constant (uniform) speed.

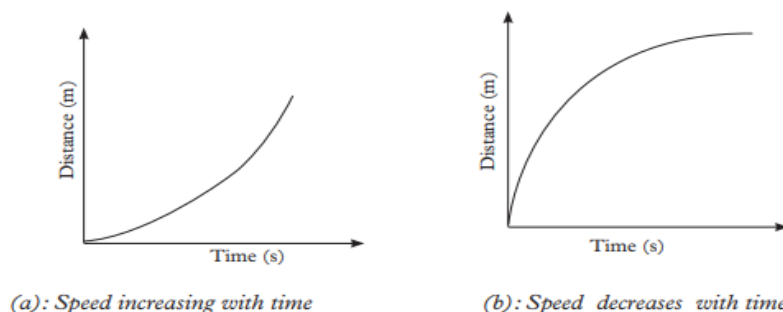


Figure 17: Distance - time graph for a body moving with positive and negative acceleration

Examples of real-life settings where such motion is exhibited include:

A body rolling down an inclined plane and a car accelerating uniformly from rest.

In the figure17 (b), the speed of the object is decreasing, implying that the object is decelerating.

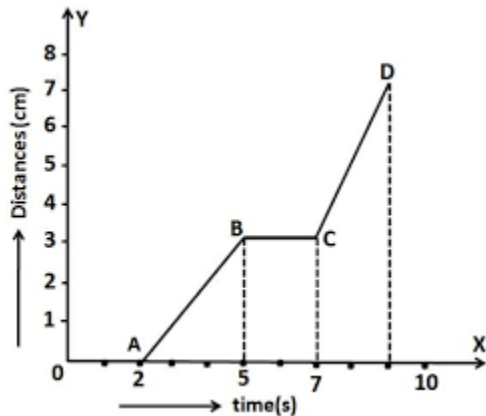
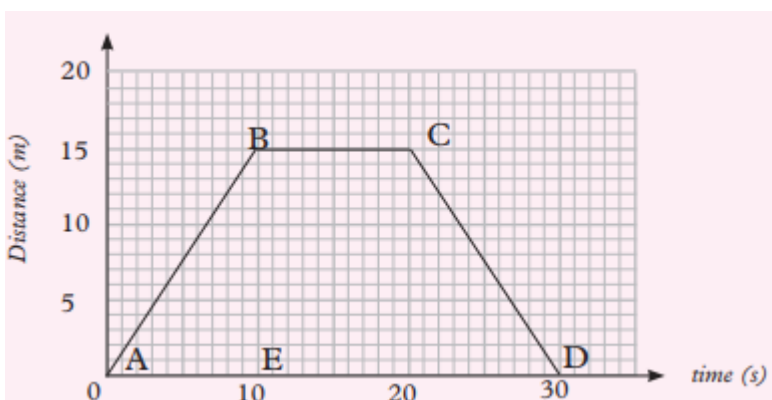


Figure 18: Displacement vs time at non constant acceleration

Examples of real life setting where such motion is exhibited include: a body thrown vertically upward, a body rolling uphill an inclined plane and a car decelerating uniformly.

### Solved Examples:

The figure below shows a distance-time graph for a motorist. Study it and answer the questions that follow.



- How far was the motorist from the starting point after 10 seconds?
- Calculate the average speed of the motorist for the first 10 seconds.
- Describe the motion of the motorist in regions (i) BC (ii) CD

## Solutions

(a) By reading directly from the graph, distance travelled in 10 s = 15 m.

(b) Slope of the graph = speed of the motorist.

$$\text{Slope} = \frac{\text{change in distance}}{\text{change in time}} = \frac{(15 - 0)\text{m}}{(10 - 0)\text{s}} = 1.5 \text{ m/s}$$

(c) (i) In the interval BC, distance is not changing but time changes, hence the body is at rest (stationary).

(ii) In the interval CD, the motorist is moving at a constant speed towards the starting point.

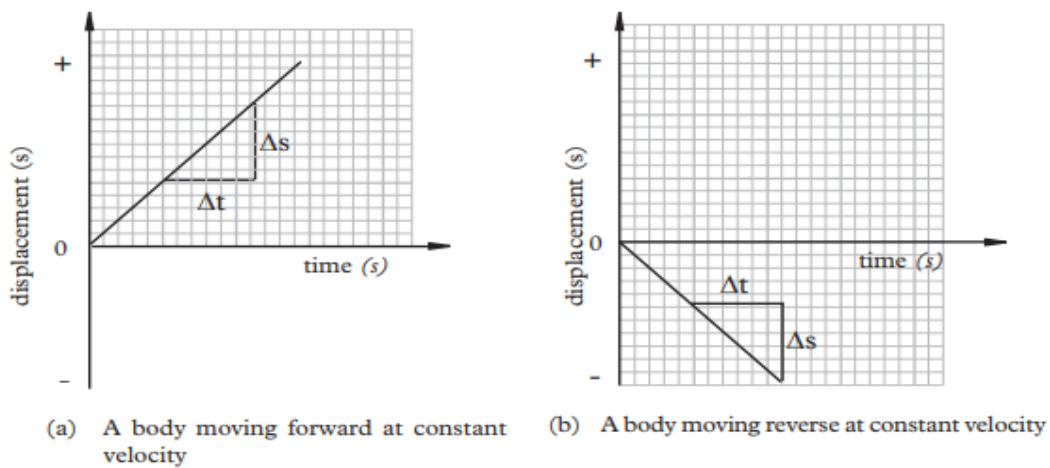


Figure 19: Speed-time graph for a body in uniform speed

### 2.2.3. Graphs of velocity vs time

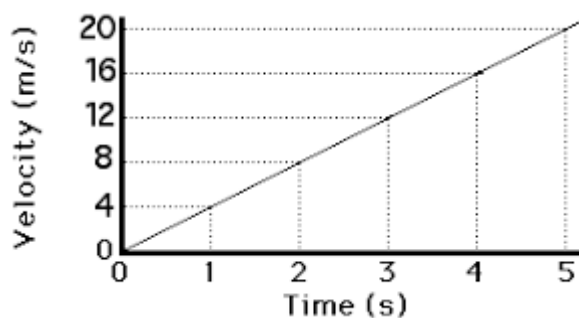


Figure 20: Velocity vs time with positive acceleration

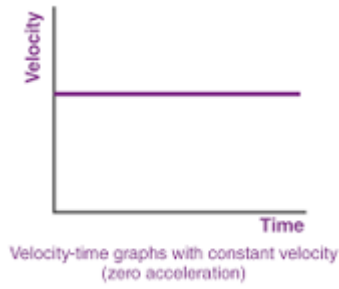


Figure 21: Velocity vs time with zero acceleration

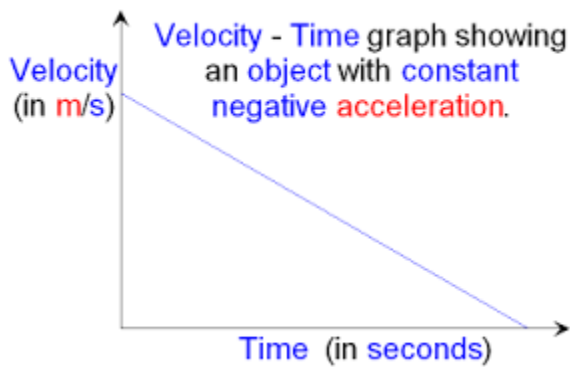


Figure 22: velocity vs time with negative acceleration



### Theoretical learning Activity

In groups of four, solve the following problems: The figure below shows a graph of speed against time for the motion of a car travelling from Musanze to Muhanga. Determine: (a) the acceleration of a car in the first 4 s. (b) the distance travelled in the first 4 s.

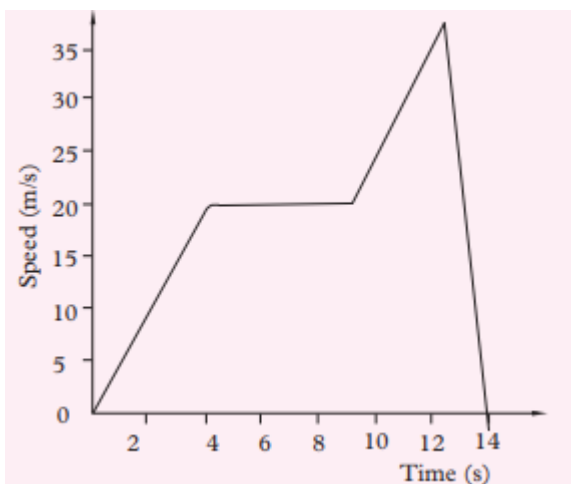


Figure 23: Velocity-time graph

1. Sketch the following graphs.

- (i) The speed-time graph for a body moving with uniform speed.
- (ii) The distance-time graph for a body moving with uniform speed.
- (iii) The speed-time graph for a body moving with non-uniform (speed) acceleration.
- (iv) The speed-time graph for a body moving with non-uniform acceleration.
- (v) The speed-time graph for a ball thrown upwards and then caught again.



### Practical learning Activity

#### Materials:

Graph papers, pencil, ruler Steps

1. Draw and interpret speed-time graphs for a body: (a) at rest.

(b) Moving with uniform and non –uniform acceleration.



## Points to Remember

### Equation of a straight line

$$Ax + By + C = 0$$

Graphs of:

- ✓ Displacement as a function of time
- ✓ Velocity as a function of time



## 2.3: Application of equations of motion

### 1.3.1. The equation of final velocity as a function of time

#### Initial velocity

The initial velocity is the velocity of the object before the effect of acceleration, which causes the change.

#### Final velocity

To compute for **final velocity**, three essential parameters are needed and these parameters are initial velocity ( $u$ ), acceleration ( $a$ ) and time ( $t$ ).

When the velocity of a body is changing, the body is said to be accelerating. Acceleration is defined as the rate of change of velocity with time.

$$v = u + at$$

Acceleration will be calculated as  $a = \frac{v-u}{t}$

### 1.3.2. Displacement as a function of time

For an object moving with constant acceleration, the average velocity is equal to the average of the initial velocity and final velocity;

$$v_{avg} = \frac{(v+u)}{2} \quad (1)$$

To find an expression for the displacement in terms of the initial and final velocity, we can set the expressions for average velocity equal to each other:

$$v_{avg} = \frac{\Delta s}{\Delta t} = \frac{(v+u)}{2} \quad (2)$$

Multiplying both sides of the equation by  $\Delta t$  leaves us with an expression for the displacement of any object moving with constant acceleration:

$$\Delta S = \frac{(v+u)}{2} \Delta t \quad (3)$$

And final velocity is given by

$$v = u + a\Delta t$$

Substituting

$$v = u + a\Delta t \text{ into } \Delta S = \frac{(v+u)}{2} \Delta t \text{ gives: } \Delta S = u\Delta t + \frac{1}{2} a\Delta t^2$$

### Example 1.

A race car reaches a speed of 42m/s. It immediately then begins a uniform negative acceleration, using its braking system, and comes to rest 5.5s later. Find how far the car moves while stopping.

#### Answer:

Use the equation for displacement;

$$\Delta S = (u + v)/2) \Delta t$$

$$= (0 + 42)/2 \times 5.5 = 115.5\text{m}$$

### Example 2.

A plane starting at rest at one end of a runway undergoes a constant acceleration of 4.8m/s<sup>2</sup> for 15s before take-off. What is its speed at take-off? How long must the runway be for the plane to be able to take off?

#### Answer

Use the equation for the velocity of a constantly accelerated object:

$$v = u + a\Delta t = (0 + 4.8 \times 15) \text{ m/s} = 72\text{m/s}$$

Use the equation for the displacement:

$$\Delta S = u\Delta t + \frac{1}{2} a (\Delta t)^2 = [0 + \frac{1}{2} \times 4.8 \times (15)^2] \text{ m} = 540\text{m}$$

### 1.3.3. Analyze freely falling objects



Figure 24: Motion due to gravity.

In Physics, free fall refers to the motion of an object under the influence of gravity only, without any other forces acting on it. In a vacuum, where air resistance is negligible, all objects near the surface of the Earth experience the same acceleration due to gravity, regardless of their mass.

**Acceleration due to Gravity ( $g$ ):** Near the surface of the Earth, the acceleration due to gravity is approximately  $9.8 \text{ m/s}^2$ . This means that in the absence of other forces, an object in free fall will experience an acceleration of  $9.8 \text{ m/s}^2$  downward.

#### Example 1.

Let's say you are standing next to a cliff and decide to drop a ball. What is the ball's velocity after 4s?

**Answer:**

$$\begin{aligned} \text{From } v &= u - gt \\ v &= u - gt \\ &= 0 - 9.81 \text{ m/s} \times 4 \text{ s} \\ &= -39.32 \text{ m/s} \end{aligned}$$

The negative sign shows this – the motion is downwards.

### Example 2.

A stone dropped from the top of a building takes 6s to reach the ground below. a) What is the height of the building? b) How far will the stone fall during the fifth second of its falling?

**Answer:**

a) Using the equation  $h = ut - \frac{gt^2}{2}$

$$h = 0 - \frac{10\text{m/s}^2 \times (6\text{s})^2}{2}$$
$$= \frac{360\text{ m}}{2}$$
$$= 180\text{m}$$

The negative sign indicates that the height of the building was calculated from the point of release of the stone downwards to the bottom of the building.

b) The fifth second begins immediately after the end of the fourth second and stops at the end of the fifth second.

$$\text{For } t = 4\text{s}, h_1 = ut - \frac{gt^2}{2}$$
$$= 0 - \frac{10 \times (4)^2}{2}$$
$$= -80\text{m}$$
$$\text{for } t = 5\text{s}, h_2 = ut - \frac{gt^2}{2}$$
$$= 0 - \frac{10 \times (5)^2}{2}$$
$$= -125\text{m}$$

The height the stone falls through in the fifth second is then given by:

$$\Delta h = h_2 - h_1$$
$$= -125 - (-80)$$
$$= -45\text{m}.$$

The negative sign indicates that measurement was from up to down, following the motion of the stone.

**Note:** An object thrown downward or upward at a given location on the Earth and in the absence of air resistance, all objects fall with the same place acceleration. (Equator), (pole)

### Example 1.

A man fires a stone out of a slingshot directly upwards. The stone has an initial velocity of 15m/s. How long will it take for the stone to return to the level he fired it at?

**Answer:**

Using the equation

$$h = ut - \frac{gt^2}{2} \text{ we have: } h = ut - \frac{gt^2}{2}$$

$$0 = 15t - \frac{10t^2}{2}$$

$$\frac{10t^2}{2} = 15t$$

$$\frac{10t}{2} = 15s$$

$$10t = 30s$$

$$t = 3s$$

### Representing Free Fall by Graphs

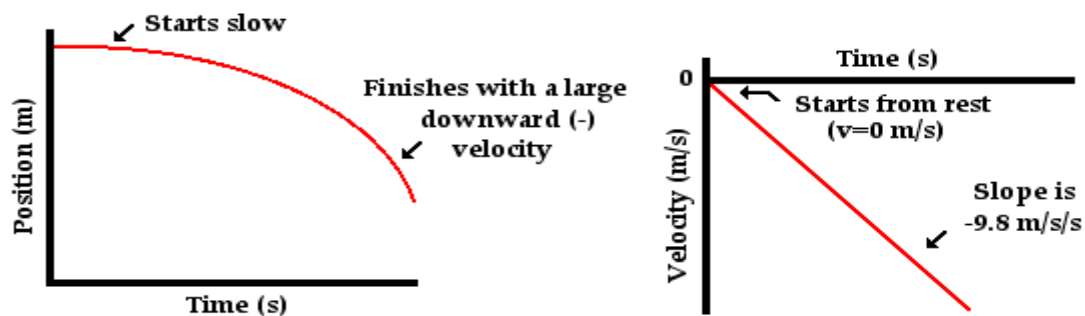


Figure 25: Dropped objects graphs



### Theoretical learning Activity

In groups of four, solve the following problems

1. Jason hits volleyball so that it moves with an initial velocity of 6.0m/s straight upward. If the volley starts from 2.0m above the floor, how long will it be in the air before it strikes the floor? Assume that Jason is the last player to touch the ball before it hits the floor.
2. A stone is thrown upwards with an initial speed of 5m/s. a) what will its maximum height be? b) When will it strike the ground? c) Where will it be in 2s?
3. Suppose that a ball is dropped from a tower 70m high. How far will it have fallen after 1s, 2s, and 3s? Assume  $y$  is positive downward. Neglect air resistance.



## Practical learning Activity

### Investigating Free Fall Acceleration

**Objective:** To experimentally determine the acceleration due to gravity by analyzing the motion of objects in free fall.

#### Materials Needed:

3. Stopwatch or timer
4. Ruler or measuring tape
5. Small, dense objects (e.g., small balls, coins, or paperweights)
6. A tall and open space (like a staircase or an outdoor area with no obstructions)
7. Lab notebook and writing materials

#### Procedure:

1. **Preparation:** a. Select a location where you can drop objects freely without obstacles and where you can measure the height of the fall. b. Gather the materials needed.
2. **Experimental Setup:** a. Measure and record the height from which you will drop the objects. Make sure the drop height is significant but manageable for accurate timing. b. Use the ruler or measuring tape to measure the exact height.
3. **Selection of Objects:** a. Choose different objects of various masses. Ensure that they are small and dense to minimize air resistance. b. Record the mass of each object.
4. **Data Collection:**
  - a. Hold one of the objects at the selected height and release it without imparting any additional force.
  - b. Use the stopwatch to measure the time it takes for the object to fall from the release point to the ground.
  - c. Record the time in your lab notebook.

**5. Repeat the Experiment:**

- a. Repeat the procedure for each selected object multiple times to obtain an average time for each.
- b. Ensure that you drop the objects from the same height consistently.

**6. Analysis:**

- a. Calculate the average time of fall for each object.
- b. Use the kinematic equation  $d = \frac{1}{2}gt^2$  to calculate the gravitational acceleration ( $g$ ) for each trial.

**7. Results and Conclusion:**

- a. Compare the calculated values of  $g$  for different objects.
- b. Discuss any variations and potential sources of error.
- c. Conclude whether the experimental results support the expected value of  $g$  (approximately  $9.8\text{m/s}^2$ ).

**8. Extension:**

- a. Explore how the mass of the objects affects the free fall acceleration.
- b. Consider the impact of changing the drop height on the results.

**9. Reporting:**

- a. Prepare a report summarizing the experiment, including the objective, materials, procedure, results, analysis, and conclusions.



## Points to Remember

- **The initial velocity** is the velocity of the object before the effect of acceleration, which causes the change.
- $a = \Delta v / \Delta t = (v_f - v_i) / (t_f - t_i)$
- Average velocity  $A_v v = (v + u) / 2$
- Displacement  $\Delta S = u\Delta t + 1/2 a (\Delta t)^2$



## Learning outcome 2 Formative Assessment

### Written assessment

1. a) Is an object accelerating if its speed is constant?

b) Is an object accelerating if its velocity is constant?

#### Solution

a) No, if the speed of an object is constant, it means that the object is not accelerating.

b) No, if the velocity of an object is constant, then the object is not accelerating.

2. If you know the position vectors of a particle at two points along its path and also know the time it took to get from one point to the other, explain how you will determine the particle's instantaneous velocity and its average velocity.

#### Solution

##### **Instantaneous Velocity:**

Calculate the derivative of the position vector  $r(t)$  with respect to time ( $dr/dt$ ) to get the instantaneous velocity vector at a specific point.

##### **Average Velocity:**

- Determine the position vectors  $r_1$  and  $r_2$  at the initial and final points.
- Calculate the displacement vector  $\Delta r = r_2 - r_1$ .

- Divide the displacement vector by the time interval ( $\Delta t$ ) to get the average velocity vector  $V_{\text{avg}} = \Delta r / \Delta t$ .
2. The average velocity of a particle moving in one dimension has a positive value. Is it possible for the instantaneous velocity to have been negative at any time in the interval? Suppose the particle started at the origin  $x = 0$ . If its average velocity is positive, could the particle ever have been in the  $-x$  region of the axis?

**Solution**

- Average velocity is a measure of overall displacement over a time interval.
  - Instantaneous velocity can be negative at some points during the interval if the particle changes direction briefly.
  - The particle could have been in the  $-x$  region momentarily, but its overall motion (average velocity) is in the positive  $x$ -direction.
3. If the average velocity of an object is zero in some time interval, what can you say about the displacement of the object for that interval?

**Solution**

If the average velocity is zero over a certain time interval, the object's displacement for that interval is zero, indicating that the object has returned to its initial position.

4. Can the magnitude of the instantaneous velocity of an object ever be greater than the magnitude of its average velocity?

**Solution**

Yes, it is possible for the magnitude of the instantaneous velocity of an object to be greater than the magnitude of its average velocity.

The magnitude of the instantaneous velocity can be greater than the magnitude of the average velocity when the object experiences variations in its speed throughout the time interval.

5. If the average velocity is non-zero for some time interval, does this mean that the instantaneous velocity is never zero during this interval? Explain.

### **Solution**

Not necessarily. The average velocity being non-zero over a time interval does not guarantee that the instantaneous velocity is never zero during that interval.

Non-zero average velocity doesn't imply that the instantaneous velocity is never zero during the interval. The object may experience moments of rest or have instantaneous velocities of zero even if the average velocity is non-zero.

6. If the velocity of a particle is non-zero, can its acceleration be zero? Explain.

### **Solution**

Yes, the velocity of a particle can be non-zero while its acceleration is zero. Acceleration is the rate of change of velocity with respect to time. If the velocity of a particle is constant, meaning it is not changing in magnitude or direction, then the acceleration is zero.

7. A stone is thrown vertically upward from the top of a building. Does the stone's displacement depend on the location of the origin of the coordinate system? Does the stone's velocity depend on the origin? (Assume that the coordinate system is stationary with respect to the building). Explain.

### **Solution**

The stone's displacement depends on the origin of the coordinate system, but its velocity does not. Displacement is affected by the choice of reference point, while velocity is independent of that choice, reflecting the stone's motion and direction.

8. If the velocity of a particle is zero, can its acceleration be nonzero? Explain.

### **Solution**

Yes, the velocity of a particle can be zero while its acceleration is non-zero. Acceleration is the rate of change of velocity with respect to time. If the velocity is zero at a specific instant, it means the object is momentarily at rest. However, the acceleration can be non-zero if the object is experiencing a change in velocity, even though its instantaneous velocity is currently zero.

9. Two cars are moving in the same direction in parallel lanes along a highway. At some instant, the velocity of car, A exceeds the velocity of car, B. Does this mean that if the acceleration of the car reduces, A is greater than that of car, B? Explain.

**Solution**

No, the fact that the velocity of car A currently exceeds the velocity of car B does not necessarily imply that if the acceleration of car A reduces, it will be greater than that of car B. The relationship between velocity, acceleration, and their changes can be more complex.

11. You are standing on top of a cliff and you decide to throw a stone upward at a speed of. After 40s, you see the stone hit the base of the cliff. How far down is the base of the cliff? In addition, what is the velocity of the stone when it reaches the base of the cliff?

**Solution**

**1. Calculate the Distance ( $d$ ):**

- Using the kinematic equation:

$$d = v_0t + \frac{1}{2}at^2$$

$$d = 0 \cdot 40 + \frac{1}{2}(-9.8) \cdot (40)^2$$

$$d = -\frac{1}{2} \cdot 9.8 \cdot 1600$$

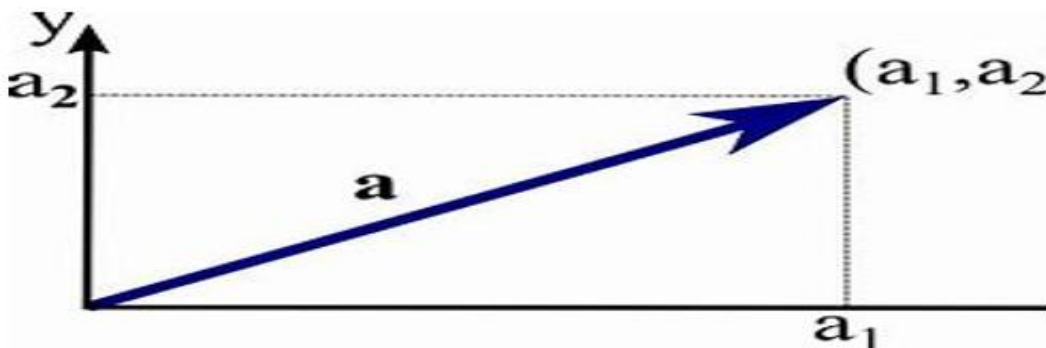
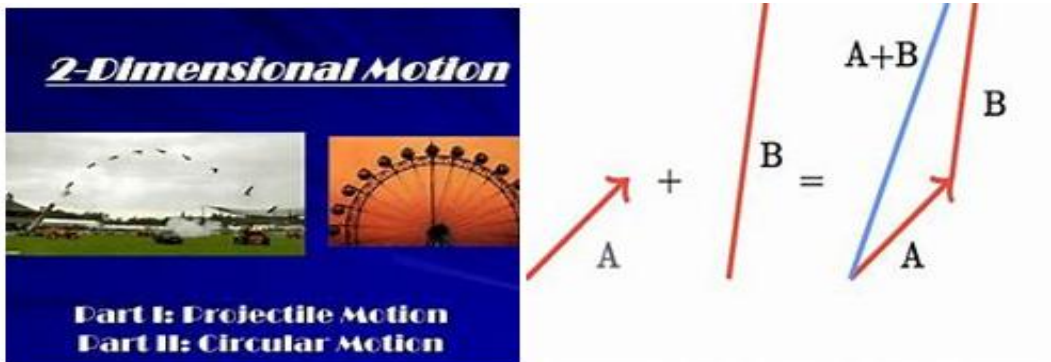
**2. Calculate the Final Velocity ( $v$ ):**

- Using the kinematic equation:

$$v = v_0 + at$$

$$v = 0 - 9.8 \cdot 40$$

### Learning Outcome 3: Analyze Motion in Two Dimensions



### Learning outcome 3. Analyze motion in two Dimension

#### Indicative contents:

- 3.1. Description of scalars, vectors and vector components
- 3.2. Illustration of displacement, velocity and acceleration
- 3.3. Analysing motion



Duration: 6 hours



### Learning outcome 3 objectives:

By the end of the learning outcome, the trainees will be able to:

1. Describe clearly scalars, vectors and vector components in Cartesian coordinate system
2. Illustrate clearly displacement, velocity and acceleration concepts in 2 dimensions
3. Analyse critically motion in two dimensions and projectile motion



### Resources

Equipment	Tools	Materials
- PPE, Whiteboard and chalkboard, computer, projector and textbooks	- Scientific calculator, meter ruler, compass	- Chalks, Markers



### Advance preparation:

- Prepare a simulation from YouTube showing projectile motion as an example of motion in two dimensions. Prepare it in advance to be shown to trainees.



## 3.1 : Description of scalars, vectors and vector components

The motion of a particle in a plane is called 2-dimensional motion.

Example: An insect crawling on your laptop or mobile screen but not flying above it.

### 3.1.1. Scalars

A scalar quantity is described completely by magnitude or numbers alone. Examples of scalar quantities are **length, mass, distance, energy, volume**, etc. A vector quantity needs a magnitude as well as a direction to describe it completely. Examples of vector quantities are displacement, velocity, weight, dipole moment, etc.

#### Magnitude

In mathematics, the magnitude or size of a mathematical object is a property which determines whether the object is larger or smaller than other objects of the same kind.

#### Properties

**Property 1:** The magnitude values of the scalar quantity are absolute  $|a| |b| = ab$ .

**Property 2:** Scalar quantity adheres to the associative property  $a (bc) = (ab) c$ .

**Property 3:** Scalar quantity follows the rule of commutative property  $ab = ba$ .

**Property 4:** Distributive property is also part of the Scalar quantity  $a (b + c) = ab + ac$ .

**Property 5:** The unity rule of identity property is seen in Scalar Quantity  $1a = a$ .

#### Property 6:

- If a Scalar quantity is with a negative sign, then according to the multiplicative property, the sign also multiplies with the term  $(-1) a = -a$ .

- If a Scalar quantity is multiplied with '0', the term also nullifies in accordance to the multiplicative property  $(0)a = 0b$ .

**Example 1:** George walks to school every day. The distance between his school and home is 500m. He takes a bus from the bus stand, which is 150m from his school, on the way back home. What is the distance between his home and the bus stand?

**Solution 1:**

**Given:**

Total distance from home to school,  $DT = 500\text{m}$

Distance between bus stand and school,  $D1 = 150\text{m}$

**To Find Out:**

Distance between home and bus stand,  $D2 = ?$

In words,

To calculate the distance between home and the bus stand, you will need to subtract the total distance from home to school from the distance between the bus stand and school.

In figures

$$D2 = DT - D1$$

$$= 500 - 150$$

$$= 350\text{m}.$$

So, the distance between George's home and the bus stand is 350m. No direction is required to calculate the answer; hence Distance is a Scalar Quantity.

### 3.1.2. Vectors

A **vector** is any quantity that has both **magnitude** and **direction**. Two examples of vectors are force and velocity. Both force and velocity are in a particular direction. The magnitude of the force indicates the strength of the force. For velocity, the speed is the magnitude. Other examples include displacement, acceleration. Note that magnitude and direction are the two properties of a vector. Geometrically, we represent a vector as a directed line segment, whose length is proportional to the and with an arrow indicating the direction.



Figure 26: Vector representation.

### Notation of vectors

We denote vectors using different ways.

- i) Bold capital letters e.g. **AB**
- ii) Capital letters with arrows e.g.  $\vec{AB}$
- iii) Position vectors with bold and small letters e.g. **a** or **b** or  $\vec{a}$  or  $\vec{b}$ .

### The Vector Product

The cross product of two vectors is equal to the product of the magnitude of the two given vectors and sine of the angle between these vectors. The vector product is represented as

$$\mathbf{A} \times \mathbf{B} = |\mathbf{A}| |\mathbf{B}| \sin \theta$$

Where,

A and B are two vectors

$|\mathbf{A}|$  = magnitude of vector A

$|\mathbf{B}|$  = magnitude of vector B

$\theta$  = angle between the vectors A and B

**Some properties of the vector product are discussed below:**

- The cross-product follows the ant-commutative law. This means it does not obey the commutative property.

$$\mathbf{A} \times \mathbf{B} \neq \mathbf{B} \times \mathbf{A}$$

$$\text{But, } \mathbf{A} \times \mathbf{B} = (-\mathbf{B}) \times \mathbf{A}$$

- It follows the distributive property.

$$\mathbf{A} \times (\mathbf{B} + \mathbf{C}) = \mathbf{A} \times \mathbf{B} + \mathbf{A} \times \mathbf{C}$$

- When the vectors are perpendicular to each other, then the vector product is maximum.
- Due to parallel and anti-parallel vectors, the cross product becomes zero.
- When a vector gets multiplied by itself, then it results in a zero vector.
- The orthogonal unit vectors show the cross product in the following manner,

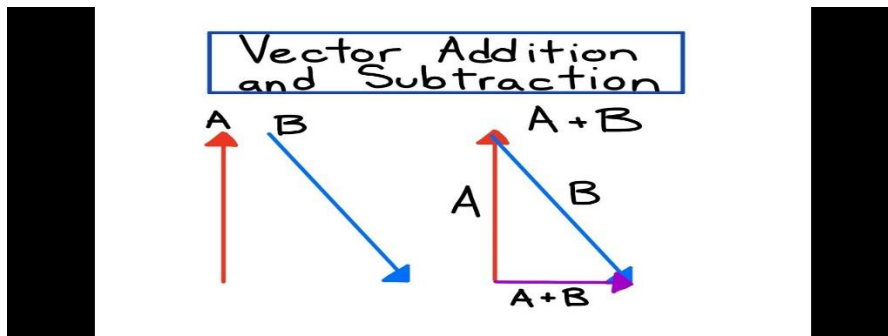
$$\mathbf{i} \times \mathbf{i} = \mathbf{j} \times \mathbf{j} = \mathbf{k} \times \mathbf{k} = 0$$

$$\mathbf{i} \times \mathbf{j} = \mathbf{k}, \mathbf{j} \times \mathbf{k} = \mathbf{i}, \mathbf{k} \times \mathbf{i} = \mathbf{j}$$

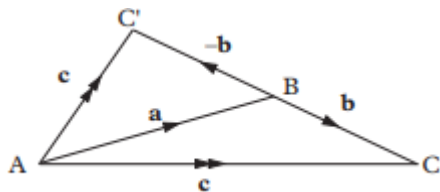
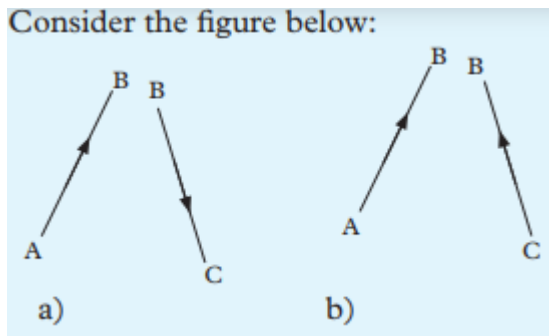
$$\mathbf{j} \times \mathbf{i} = -\mathbf{k}, \mathbf{k} \times \mathbf{j} = -\mathbf{i}, \mathbf{i} \times \mathbf{k} = -\mathbf{j}$$

*Operations on vectors*

## Addition and subtraction of vectors by construction



Consider the figure below:



In the figure above, the end result of moving from A to B and then from B to C is the same as going from A to C directly. The end effect is to reach point C from point A. Since the end result is same, we write  $\mathbf{AC} = \mathbf{AB} + \mathbf{BC}$   $\mathbf{c} = \mathbf{a} + \mathbf{b}$

The vector  $\mathbf{AC}$  is called the **resultant vector** and is indicated by the double arrow.

Similarly, if you go from A to B and then from B to  $C'$ , the effect is the same as going from A to  $C'$  directly. The required effect is to reach point  $C'$  from point A.

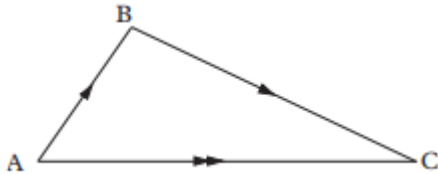
Since the end result is the same, we write  $\mathbf{AC}' = \mathbf{AB} + \mathbf{BC}'$

$$\mathbf{c} = \mathbf{a} + \mathbf{-b}$$

$$\mathbf{c} = \mathbf{a} - \mathbf{b}$$

The vector  $AC'$  is called the resultant vector and is indicated by the double arrow. It is also important to note that:  $AC' = AB - BC'$

Let us consider another triangle ABC in figure below. The triangle represents routes joining three towns A, B and C.



If you go from A to B, then from B to C, the effect is the same as going from A to C directly.

The required effect is to reach town C from A.

Since the effect is the same, then  $AB + BC = AC$ .

Vector  $AC$  is called the **resultant vector** of  $AB$  and  $BC$ . Such a vector is usually represented by a line segment with a double arrowhead.

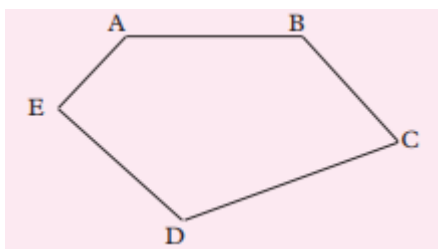
### Example

Using Fig. below, write down the single vector equivalent to:

(a)  $AB + BC$     (b)  $AE + ED$     (c)  $BC + CD + DE$     (d)  $ED + DC + CB$     (e)  $AB + BA$

(f)  $CD + DC$     (g)  $AE + EB + BC$     (h)  $CD + DE + EB$     (i)  $AB + BC + CD + DE$

(j)  $DE + EA + AB + BC + CD$



### Solution

- (a)  $AB + BC = AC$  (Moving from  $A$  to  $B$ , then from  $B$  to  $C$  is equivalent to moving from  $A$  to  $C$  directly.)
- (b)  $AE + ED = A \text{ to } E \text{ then to } D.$   
 $= A \text{ to } D$   
 $= AD$
- (c)  $BC + CD + DE = BD + DE = BE$
- (d)  $ED + DC + CB = EC + CB = EB$
- (e)  $AB + BA = AB - AB = 0$
- (f)  $CD + DC = 0$  (from  $A$  to  $B$  then back to  $A$ )
- (g)  $AE + EB + BC = AB + BC = AC$
- (h)  $CD + DE + EB = CE + EB = CB$
- (i)  $AB + BC + CD + DE = AC + CD + DE = AD + DE = AE$
- (j)  $DE + EA + AB + BC + CD$   
 $= DA + AB + BC + CD$   
 $= DB + BC + CD$   
 $= DC + CD$   
 $= 0$  (Start from  $D$  and back to  $D$ )

### What is Vector Addition?

The process of adding two or more vectors is called vector addition. Depending on the direction of the vector, vector addition is categorized into two types. They are –

- Parallelogram law of vector addition
- Triangular law of vector addition



The method of vector addition is chosen based on the arrangement of the head and tail of vectors.

- If two vectors are arranged head to tail the triangular law of vector addition is followed.
- If two vectors are arranged head-to-head or tail to tail then, the parallelogram law of vector addition is followed.

## Parallelogram law of vector addition

If two vectors are arranged head-to-head or tail to tail then, the parallelogram law of vector addition is carried out.

### Statement

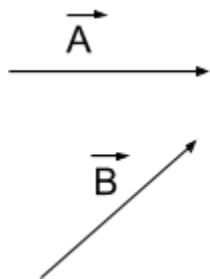
“If two vectors are considered to be the adjacent sides of a parallelogram, then the resultant of two vectors is given by the vector that is a diagonal passing through the point of contact of two vectors.”

### Method

Step-wise vector addition of two vectors using Parallelogram law of vector addition is given below-

#### Step 1:

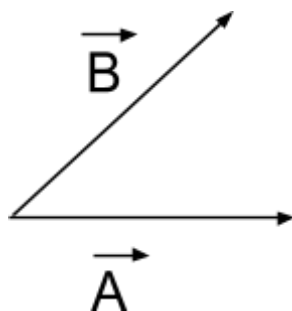
Consider two vectors; A and B



#### Step 2:

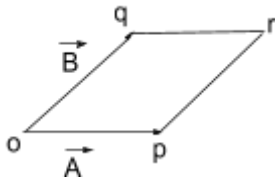
Bring the tail of A to the tail of B

Here the direction of vectors is not changed.



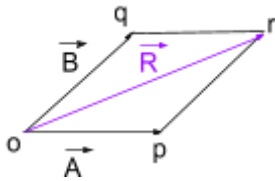
**Step 3:**

Draw a line parallel to A and B with the same magnitude, in a way to complete parallelogram.



**Step 4:**

Join the point o and r by a straight line with an arrow pointing towards the r. This is diagonal to the parallelogram.



And this is the resultant vector  $A+B=R$

Triangular law of vector addition.

If two vectors are arranged head to tail the triangular law of vector addition is carried out.

**Statement:**

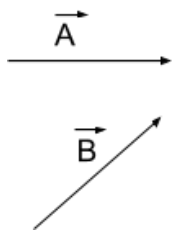
“When two vectors are represented by two sides of a triangle in magnitude and direction were taken in the same order then the third side of that triangle represents in magnitude and direction the resultant of the vectors.”

**Method**

Step-wise vector addition of two vectors using Triangular law of vector addition is given below-

**Step 1:**

Consider two vectors; A and B



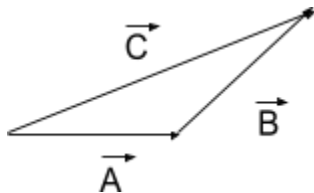
**Step 2:**

Bring the head of A to the tail of B Here the direction of vectors is not changed.



**Step 3:**

Join the tail of A to the head of B by a straight line with an arrow pointing towards the head of B



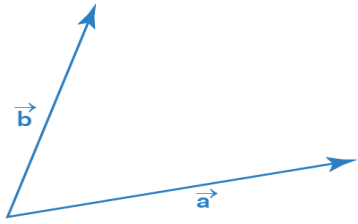
This new vector is the resultant vector  $A+B=C$

**Vector Subtraction by Parallelogram Law**

Suppose that **a** and **b** are two vectors. How can we interpret the subtraction of these vectors graphically? That is, what meaning do we attach to **a - b**? To start with, we note that **a - b** will be a vector which when added to **b** should give back **a**. i.e.,  $(\mathbf{a} - \mathbf{b}) + \mathbf{b} = \mathbf{a}$

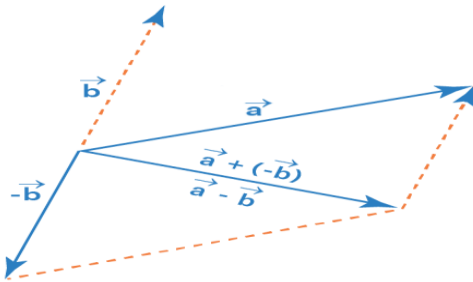
But how do we determine the vector **a - b**, given the vectors **a** and **b**?

The following figure shows vectors **a** and **b** (we have drawn them to be co-initial).



Using the parallelogram law of vector addition, we can determine the vector as follows. We interpret  $\mathbf{a} - \mathbf{b}$  as  $\mathbf{a} + (-\mathbf{b})$ , that is, the vector sum of  $\mathbf{a}$  and  $-\mathbf{b}$ . Now, we reverse vector  $\mathbf{b}$ , and then add  $\mathbf{a}$  and  $-\mathbf{b}$  using the parallelogram law:

Vector Subtraction by Parallelogram Law

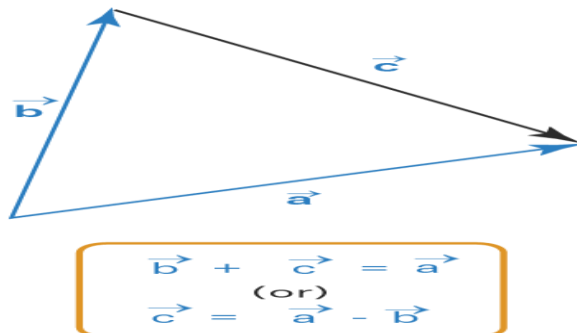


This shows the vector subtraction  $\mathbf{a} - \mathbf{b}$  as the addition of  $\mathbf{a}$  and  $-\mathbf{b}$ .

### Vector Subtraction by Triangle Law

Now, we will interpret the subtraction of vectors using the triangle law of vector addition. Denote the vector drawn from the end-point of  $\mathbf{b}$  to the end-point of  $\mathbf{a}$  by  $\mathbf{c}$ .

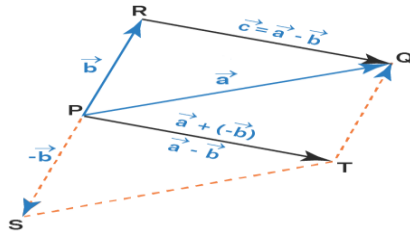
Vector Subtraction by Triangle Law



Note that  $\mathbf{b} + \mathbf{c} = \mathbf{a}$ . Thus,  $\mathbf{c} = \mathbf{a} - \mathbf{b}$ . In other words, the vector  $\mathbf{a} - \mathbf{b}$  is the vector drawn from the tip of  $\mathbf{b}$  to the tip of  $\mathbf{a}$  (if  $\mathbf{a}$  and  $\mathbf{b}$  are co-initial).

Note that both ways (using parallelogram law and triangle law) are described above give us the same vector for  $\mathbf{a} - \mathbf{b}$ . This becomes clearer from the figure below:

Vector Subtraction by Parallelogram and Triangle Laws 



The vector  $\mathbf{PT}$  is obtained by adding  $\mathbf{a}$  and  $-\mathbf{b}$  using the parallelogram law. The vector  $\mathbf{RQ}$  is obtained by drawing the vector from the tip of  $\mathbf{b}$  to the tip of  $-\mathbf{a}$ . Clearly, both vectors are the same (as their magnitudes and directions are the same).

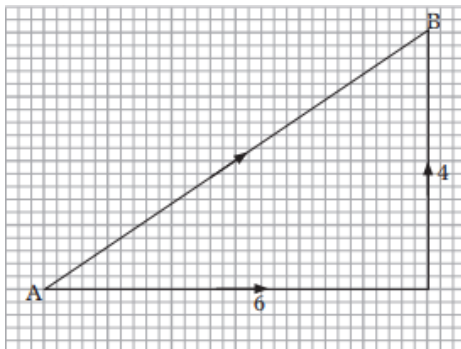
### 3.1.3. Vector components in Cartesian coordinate system

#### Definition of a column vector

Given the points A (-3, 2), B (3, 5), C (0, -2), D (2, 2) and E (-3, -3). Plot them on a graph paper. Join the points appropriately to show the following vectors.

- a) **AB**      b) **BC**      c) **CA**
- d) **ED**      e) **DA**      f) **EB**

Consider the vectors from point A to B in the figure below

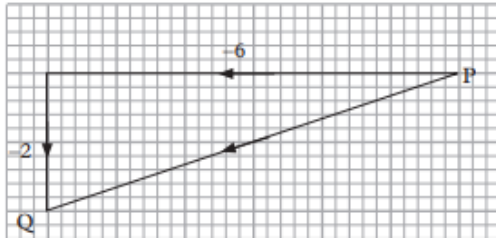


The vector **AB** and is a displacement of 6 units to the right and 4 units upwards.

Therefore, vector AB in Fig. above is represented as a column vector as

$$\vec{\mathbf{AB}} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$$

The figure below shows a displacement from P to Q.



The vector **PQ** is a displacement of 6 units to the left and 2 units downwards.  $-6$

### Operation on vectors

It is important to note that whenever the displacement is towards the right or upwards, it is a positive displacement while displacement to the left or downwards is negative.

In general, if **P** ( $x_1, y_1$ ) and **Q** ( $x_2, y_2$ ),

Then  $\mathbf{PQ} = \mathbf{OQ} - \mathbf{OP}$

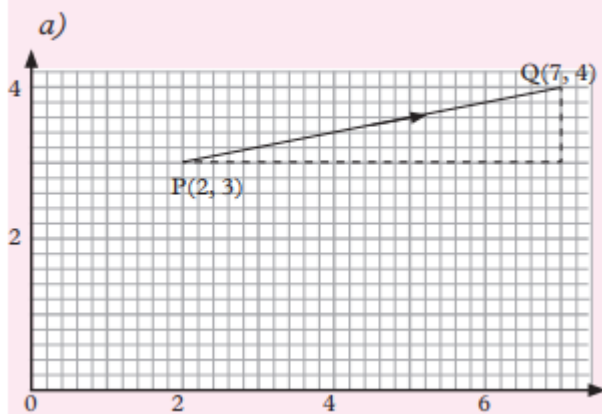
$$\mathbf{PQ} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} - \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix}$$

### Example

*Plot the points P (2, 3) and Q (7, 4) and show vector **PQ**.*

*Write down the column vector **PQ**.*

### Solution



b)  $PQ = \begin{pmatrix} 7 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$



### Theoretical learning Activity

In pairs, Draw a Cartesian plane and locate the vectors provided. Draw a line joining the two points and indicate the arrow showing the direction from the first to the last point.

A (2,2) and B (3,1)

P (5,3) and Q (2,2)

P (3,1) and Q (2, -3)

A (3,4) and B (4,10)

A (2, -3) and B (6,7)

A (5,1) and B (2, -3)

Without drawing, find the resultant vectors for the following points.

A (2,0) and B (3, -11)

P (5,1) and Q (2,4)

P (-6,1) and Q (6, -3)

A (3,2) and B (4, -5)



### Practical learning Activity

In pairs, roughly estimate the following:

The distance between your school and the nearest shopping centre. The direction of your school from the nearest shopping centre.



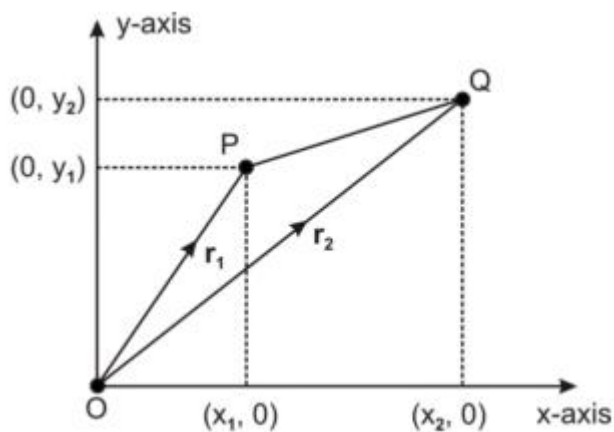
### Points to Remember (Take home message)

A **vector** is any quantity that has both **magnitude** and **direction**  
A **scalar** quantity is described completely by magnitude or numbers alone.



## 3.2 : Illustration of displacement, velocity and acceleration

The concept of motion, when a particle moves along a straight line, can be used for motion in a plane. When the motion of a particle is in-plane, we generally consider the plane of motion as the x-y plane. We choose the origin at the place from where the motion starts, and then we consider motion along any two convenient mutually perpendicular directions as one-dimensional motion. Motion in two perpendicular directions is chosen as the x and y-axes.



### 3.2.1. Displacements in two dimensions

Suppose a particle moves in a plane along the curve as shown in figure. At time  $t_1$  the particle is at point **P** and at some later time  $t_2$ , the particle is at **Q**. As the particle moves from **P** to **Q** in the time interval  $\Delta t = t_2 - t_1$ , the position vector changes from  $r_1$  to  $r_2$ . From the triangle rule of vector addition, we can write

$$\mathbf{OP} + \mathbf{PQ} = \mathbf{OQ} \Rightarrow \mathbf{PQ} = \mathbf{OQ} - \mathbf{OP} \Rightarrow \mathbf{PQ} = \mathbf{r}_2 - \mathbf{r}_1$$

$$\Delta \mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$$

Here,  $\mathbf{r}_1 = x_1\hat{i} + y_1\hat{j}$  and  $\mathbf{r}_2 = x_2\hat{i} + y_2\hat{j}$

$$\mathbf{r}_2 - \mathbf{r}_1 = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} = \Delta x\hat{i} + \Delta y\hat{j}$$

$$\Delta \mathbf{r} = \Delta x\hat{i} + \Delta y\hat{j}; \text{ where } \Delta x = (x_2 - x_1) \text{ and } \Delta y = (y_2 - y_1)$$

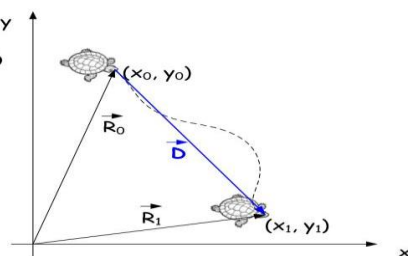
So, we see here that motion in an x-y plane can be simplified into two separate motions; one in the x-direction and one in the y-direction. Hence the strategy that we discussed to tackle the 2D motion works.

## Two-Dimensional Displacement

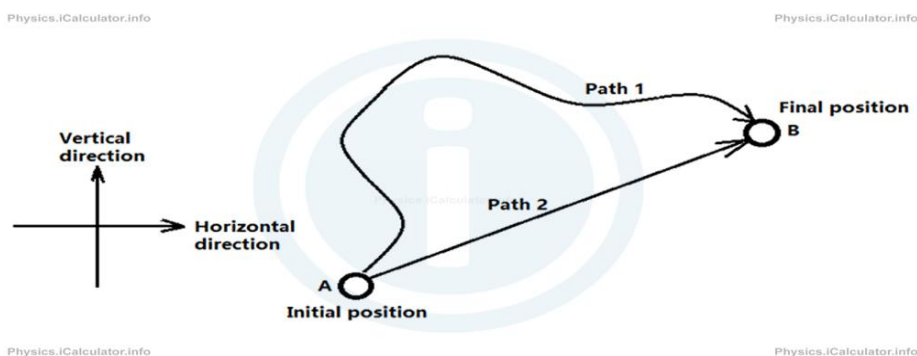
We can see the truth of this by solving for  $\mathbf{R}_1$ :

$$\vec{D} = \vec{R}_1 - \vec{R}_0 \Rightarrow \vec{R}_1 = \vec{R}_0 + \vec{D}$$

Our picture shows this addition being done, graphically.



Let's consider a visual example before dealing with numbers. Look at the figure below.



As for the displacement, it is much easier to calculate it, as we have to deal only with two components:  $AB_x$  and  $AB_y$ . Therefore, the Pythagorean Theorem is used only once, as the hypotenuse of the right triangle represents the magnitude of the displacement.

Thus, if we denote the displacement by  $\Delta\vec{r}$  we can write:

$$|\Delta\vec{r}| = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

### 3.2.2. Velocity in two dimension

We define the particle's average velocity during the time interval  $\Delta t$  as the ratios of the displacement of that time interval.

$$\mathbf{v}_{av} = \frac{\Delta\mathbf{r}}{\Delta t} = \frac{\Delta x \hat{i} + \Delta y \hat{j}}{\Delta t}$$

$$\mathbf{v}_{av} = \frac{\Delta\mathbf{r}}{\Delta t} = \frac{\Delta x}{\Delta t} \hat{i} + \frac{\Delta y}{\Delta t} \hat{j}$$

$$\lim_{\Delta t \rightarrow 0} \frac{\Delta\mathbf{r}}{\Delta t} = \lim_{\Delta t \rightarrow 0} \left( \frac{\Delta x}{\Delta t} \right) \hat{i} + \lim_{\Delta t \rightarrow 0} \left( \frac{\Delta y}{\Delta t} \right) \hat{j}$$

$$\mathbf{v}_{inst.} = v_x \hat{i} + v_y \hat{j}$$

$\mathbf{v}_{inst.}$  is instantaneous velocity.

### 3.2.3. Acceleration in two dimensions

The average acceleration of a particle as it moves from P to Q is defined as the ratio of the change in the instantaneous velocity vector

$$\mathbf{a}_{av} = \frac{\Delta\mathbf{v}}{\Delta t}$$

$$\text{and } \mathbf{a}_{inst.} = \lim_{\Delta t \rightarrow 0} \frac{\Delta\mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt}$$

$$\mathbf{a}_{inst.} = \frac{d}{dt} (v_x \hat{i} + v_y \hat{j}) = \frac{dv_x}{dt} \hat{i} + \frac{dv_y}{dt} \hat{j}$$

$$\mathbf{a}_{inst.} = \mathbf{a} = a_x \hat{i} + a_y \hat{j}$$

Because  $\mathbf{a}$  is normally assumed as constant, its component  $a_x$  and  $a_y$  are also constants. Therefore, we can apply the kinematics equations to the x and y components of the velocity vector and displacement vector as follows.

$$\mathbf{V} = \mathbf{V}_0 + \mathbf{a}t \begin{cases} v_x = (v_x)_0 + a_x t \\ v_y = (v_y)_0 + a_y t \end{cases}$$

$$\mathbf{r} = \mathbf{V}_0 + \frac{1}{2}\mathbf{a}t^2 \begin{cases} x = (v_x)_0 t + \frac{1}{2}a_x t^2 \\ y = (v_y)_0 t + \frac{1}{2}a_y t^2 \end{cases}$$

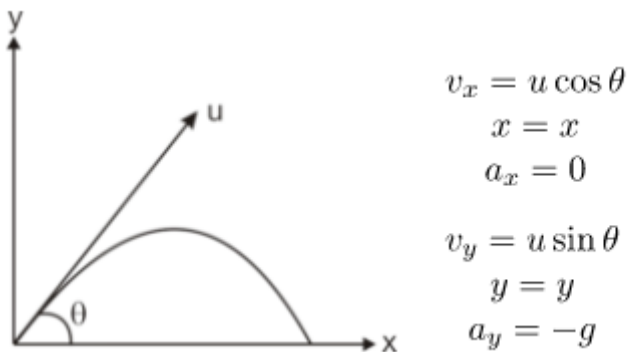
$$v^2 = u_0^2 + 2as \begin{cases} v_x^2 = (v_x)_0^2 + 2a_x s_x \\ v_y^2 = (v_y)_0^2 + 2a_y s_y \end{cases}$$

In other words, two-dimensional motion with constant acceleration is equivalent to two independent motions in the x and y directions having constant accelerations  $\mathbf{a}_x$  and  $\mathbf{a}_y$ .

### *Examples of two-dimensional motion*

#### Projectile motion

When an object is thrown at some angle to the horizontal, the projectile's motion will be in the horizontal and vertical directions. Suppose the motion of the projectile is in the x-y plane of the coordinate system. The object's motion will be along two mutually perpendicular directions together, say the x and y axes of the coordinate system. Consider motion along the horizontal direction (x-axis) and motion along the vertical direction (y-axis).



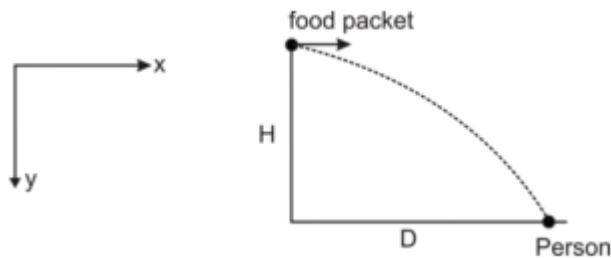
#### Solved problems on two-dimensional motion

##### **Example 1**

A helicopter on a flood relief mission flying horizontally with speed  $u$  at an altitude  $H$  has to drop a food packet for a victim standing on the ground. At what distance from the victim should the packet be dropped? The victim stands in the vertical plane of the helicopter's motion.

## Solution

We can solve this problem by using the concept involved in motion in a plane.



Suppose the velocity of the food packet at the time of release is  $v$  and is horizontal. The vertical velocity at the time of release is zero.

The motion of the food packet along the y-axis.

$$y = u_y + \frac{1}{2}a_y t^2$$

$$\text{Here, } y = H; u_y = 0; a_y = g$$

$$H = 0 + \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2H}{g}}$$

Again, the motion of the food packet along the x-axis,

$$x = u_x t + \frac{1}{2}a_x t^2$$

$$\text{Here, let } x = D; u_x = u; a_x = 0$$

$$\Rightarrow D = ut + 0 \Rightarrow D = ut$$

$$D = u\sqrt{\frac{2H}{g}}$$

Therefore, the required distance equal to

$$D = u\sqrt{\frac{2H}{g}}$$

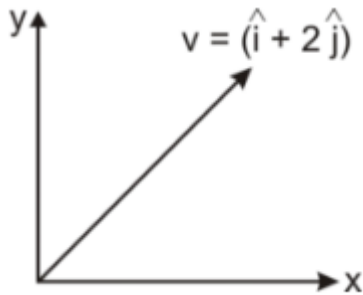
## Example 2

A projectile is given an initial velocity of  $(\hat{i} + 2\hat{j})$  m/s where  $\hat{i}$  is along the ground and  $\hat{j}$  is along the vertical of  $g=10\text{m/s}^2$  then find equation of trajectory.

## Solution

This problem is based on the concept of motion in a plane.

We can study the two-dimensional motion as one-dimensional along the x-axis and y-axis since its superposition gives the resultant motion.



$$(v_x)_i = 1 \text{ m/s}; a_x = 0$$

$$(v_y)_i = 2 \text{ m/s}; a_y = -10 \text{ m/s}^2$$

Using motion of the projectile along x-axis,

$$x = u_x t + \frac{1}{2} a_x t^2$$

$$\text{So, } x = 1 \times t + \frac{1}{2} \times (0)t^2$$

$$x = t \quad \dots(i)$$

Again, using motion of the projectile along y-axis,

$$y = u_y t + \frac{1}{2} a_y t^2$$

$$y = 2 \times t + \frac{1}{2} \times (-10)t^2$$

$$\text{So, } y = 2t - 5t^2$$

Putting the value of  $t = x$  in the above equation

$$y = 2x - 5x^2$$

This is the required equation of trajectory.

### Example 3

The particle has initial velocity  $(3\hat{i} + 4\hat{j})$  and an acceleration of  $(0.4\hat{i} + 0.3\hat{j})$ . Find speed of particle after 10 sec.

#### Solution

Here, in this question, it is given that

$$(u_x) = 3 \text{ unit}; a_x = 0.4 \text{ unit}$$

$$(u_y) = 4 \text{ unit}; a_y = 0.3 \text{ unit}$$

Since the acceleration of the particle is constant so, we can use

$$(v_x)_f = (v_x)_i + a_x t \text{ and } (v_y)_f = (v_y)_i + a_y t$$

$$(v_x)_f = 3 + (0.4 \times 10) \text{ and } (v_y)_f = 4 + (0.3 \times 10)$$

$$\text{So, } (v_x)_f = 3 + 4 \text{ and } (v_y)_f = 4 + 3$$

$$(v_x)_f = 7 \text{ unit and } (v_y)_f = 7 \text{ unit}$$

Now, we can write

$$\mathbf{v}_f = (v_x)_f \hat{i} + (v_y)_f \hat{j}$$

$$\mathbf{v}_f = (7 \text{ unit})\hat{i} + (7 \text{ unit})\hat{j}$$

$$\text{Speed of particle} = |\mathbf{v}_f|$$

$$= \sqrt{(7 \text{ unit})^2 + (7 \text{ unit})^2}$$

$$= (7 \text{ unit})\sqrt{2} = 7\sqrt{2} \text{ unit}$$

### Example 4

A particle moves in x-y plane under the action of a force  $\mathbf{F}$  such that the value of its linear momentum  $\mathbf{P}$  at any time  $t$  is  $P_x = 2 \cos t$ ;  $P_y = 2 \sin t$ . Find the angle  $\theta$  between  $\mathbf{F}$  and  $\mathbf{P}$  at a given time  $t$ .

#### Solution

Here momentum vector of the particle can be written as

$$\mathbf{P} = P_x \hat{i} + P_y \hat{j}$$

$$\mathbf{P} = 2 \cos t \hat{i} + 2 \sin t \hat{j}$$

Now, from Newton's 2<sup>nd</sup> law of motion;

$$\text{We can obtain force as } \mathbf{F} = \frac{d\mathbf{P}}{dt} \Rightarrow \mathbf{F} = \frac{d}{dt} (2 \cos t \hat{i} + 2 \sin t \hat{j})$$

$$\mathbf{F} = \frac{d\mathbf{P}}{dt} \Rightarrow \mathbf{F} = \frac{d}{dt} (2 \cos t \hat{i} + 2 \sin t \hat{j})$$

Now, using scalar product of two vector for calculation angle between  $\mathbf{F}$  &  $\mathbf{P}$ .

$$\mathbf{P} \cdot \mathbf{F} = |\mathbf{P}| |\mathbf{F}| \cos \theta$$

$$\cos \theta = \frac{\mathbf{P} \cdot \mathbf{F}}{|\mathbf{P}| |\mathbf{F}|} = \frac{(2 \cos t \hat{i} + 2 \sin t \hat{j}) \cdot (-2 \sin t \hat{i} + 2 \cos t \hat{j})}{|\mathbf{P}| |\mathbf{F}|}$$

$$\cos \theta = \frac{-4 \sin t \cdot \cos t + 4 \sin t \cdot \cos t}{\sqrt{(2 \cos t)^2 + (2 \sin t)^2} \sqrt{(-2 \sin t)^2 + (2 \cos t)^2}}$$

$$\cos \theta = 0 \Rightarrow \theta = 90^\circ$$

So, the angle between the force vector and momentum vector is  $90^\circ$ .



### Theoretical learning Activity

In groups of four, solve the following problems

**Question 1.** What is motion in 2 dimensions (2d)?

**Question 2.** Can there be motion in 2 dimensions (2d)?

**Question 3:** Find the average acceleration between  $t = 0$  and  $t = 3$ , for the particle which is moving in a plane and whose position is given below,

$$\mathbf{v} = 3t\mathbf{i} + 3t^3\mathbf{j}$$

**Question 4:** Find the average acceleration between  $t = 0$  and  $t = 2$ , for the particle which is moving in a plane and whose position is given below,

$$\mathbf{v} = t\mathbf{i} + 3t\mathbf{j}$$

**Question 5:** Find the instantaneous acceleration at  $t = 1$ , for the particle which is moving in a plane and whose position is given below,

$$\mathbf{r} = t\mathbf{i} + t\mathbf{j}$$



### Points to Remember

#### Position

$$\vec{r} = x\hat{i} + y\hat{j}$$

#### Velocity

$$\begin{aligned}\vec{v} &= \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} \\ &= \vec{v} = \frac{d\vec{r}}{dt}\end{aligned}$$

$$v = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

#### Acceleration

$$\begin{aligned}\vec{a} &= \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t} \\ &= \vec{a} = \frac{d\vec{v}}{dt}\end{aligned}$$

This can also be decomposed into its components.

$$\begin{aligned}a &= \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} \\ &= a = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j}\end{aligned}$$



## 3.3 : Analysing motion

### 3.3.1. Analyze motion in two dimensions

A particle moving in a plane can be described by using the coordinates. But often this information is not enough to fully describe the state and the behaviour of the particle. There are still some questions left unanswered. For example, how fast is it moving? In which direction and what is the acceleration and direction of acceleration of the particle? These parameters are necessary to describe the motion of the particle in a 2D plane. Fortunately, all of this can be figured out using a bit of vector algebra and calculus. Let's see how to do that in detail.

#### **Motion in a Plane**

Suppose a particle is moving from a point X on the Cartesian plane to a point Y. Position vectors are necessary to describe the current position of the particle. These vectors are always with respect to the reference frame at the origin. The following parameters are required to fully describe the behavior of a particle moving in a plane,

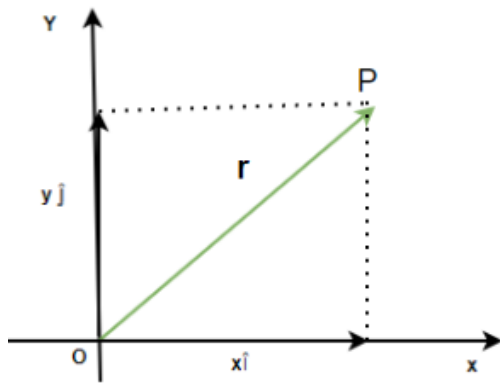
1. Position
2. Velocity
3. Acceleration

#### **Position Vector**

The vector which denotes the position and direction of the particle's position with respect to the origin is called the position vector. The position vector  $\vec{r}$  for a particle is given by,

$$\vec{r} = x\hat{i} + y\hat{j}$$

Where  $x$  and  $y$  are their components along the  $x$  and  $y$ -axis.

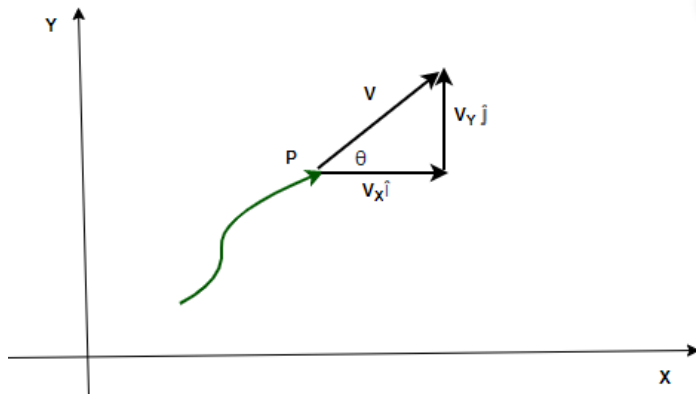


*Velocity*

The velocity of a particle can be described in two ways – average velocity and instantaneous velocity. When the particle is under acceleration, it changes its velocity every second. So, a single value cannot be assigned to a velocity. In such cases, **the instantaneous velocity** is preferred, which describes the velocity and its direction at a particular instant. It is given by,

$$\vec{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t}$$

$$= \vec{v} = \frac{d\vec{r}}{dt}$$



Velocity can also be expressed in the form of its components.

The **average velocity** is the ratio of total displacement over total time. Suppose a particle goes from  $\vec{r}$  to  $\vec{r}'$  in a total time of  $\Delta t$

The velocity is given by,

$$\vec{v} = \frac{\vec{r}' - \vec{r}}{\Delta t}$$

## Sample Problems

**Question 1:** Find the velocity at  $t = 2$ , for the particle which is moving in a plane and whose position is given below,

$$\mathbf{r} = t^2\mathbf{i} + t^2\mathbf{j}$$

**Answer:**

*Given: the initial and final position vectors,*

$$\mathbf{r} = t^2\mathbf{i} + t^2\mathbf{j}$$

*The position vector changes with time. The velocity in this case is given by the formula,*

$$\mathbf{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

*Here  $x(t) = t^2$  and  $y(t) = t^2$*

*Plugging these values into the equation,*

$$\begin{aligned} \mathbf{v} &= \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} \\ \mathbf{v} &= \frac{d}{dt}(t^2)\hat{i} + \frac{d}{dt}(t^2)\hat{j} \\ \mathbf{v} &= 2t\hat{i} + 2t\hat{j} \end{aligned}$$

*At  $t = 2$ ,*

$$\begin{aligned} \mathbf{v} &= 2t\hat{i} + 2t\hat{j} \\ &= \mathbf{v} = 2(2)\hat{i} + 2(2)\hat{j} \\ &= \mathbf{v} = 4\hat{i} + 4\hat{j} \end{aligned}$$

**Question 2:** Find the velocity at  $t = 0$ , for the particle which is moving in a plane and whose position is given below,

$$\mathbf{r} = (t+2)\mathbf{i} + (4t^2+2)\mathbf{j}$$

**Answer:**

Given: the initial and final position vectors,

$$r = (t+2)i + (4t^2+2)j$$

The position vector changes with time. The velocity in this case is given by the formula,

$$v = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

Here  $x(t) = t+2$  and  $y(t) = 4t^2+2$

Plugging these values into the equation,

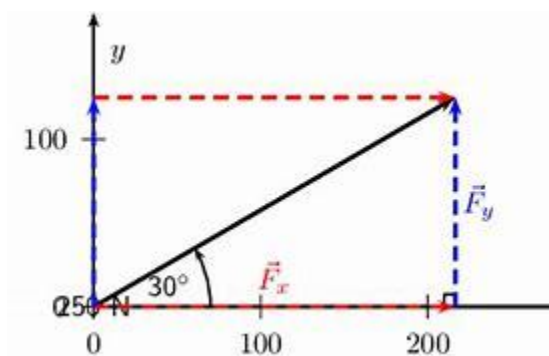
$$\begin{aligned} v &= \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} \\ v &= \frac{d}{dt}(t+2)\hat{i} + \frac{d}{dt}(4t^2+2)\hat{j} \\ v &= \hat{i} + 8t\hat{j} \end{aligned}$$

at  $t = 0$ ,

$$\begin{aligned} v &= \hat{i} + 8t\hat{j} \\ = v &= 4\hat{i} + 8(0)\hat{j} \\ = v &= 4\hat{i} \end{aligned}$$

**What are the components of a vector in two dimensions?**

The components of a vector in two-dimension coordinate system are usually considered to be x-component and y-component. It can be represented as,  $F = (F_x, F_y)$ , where F is the vector.



### 3.3.2. Projectile motion

#### Definition

Projectile motion is the **motion of an object thrown or projected into the air**, subject to only the acceleration of gravity. The object is called a projectile, and its path is called its trajectory.

One of the most common examples of motion in a plane is Projectile motion. Projectile refers to an **object that is in flight after being thrown or projected**. In a projectile motion, the only acceleration acting is in the vertical direction which is acceleration due to gravity ( $g$ ).

#### What is Projectile Motion?

When a particle is thrown obliquely near the earth's surface, it moves along a curved path under constant acceleration directed towards the centre of the earth (we assume that the particle remains close to the earth's surface). The path of such a particle is called a projectile, and the motion is called **projectile motion**.

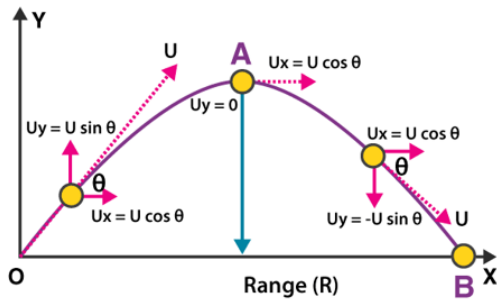
**In a Projectile Motion, there are two simultaneous independent rectilinear motions:**

1. **Along the x-axis:** uniform velocity, responsible for the **horizontal** (forward) **motion** of the particle.
2. **Along the y-axis:** uniform acceleration, responsible for the **vertical** (downwards) **motion** of the particle.

**Acceleration in the horizontal projectile motion and vertical projectile motion of a particle:** When a particle is projected in the air with some speed, the only force acting on it during its time in the air is the acceleration due to gravity ( $g$ ). This acceleration acts vertically downward. There is no acceleration in the horizontal direction, which means that the velocity of the particle in the horizontal direction remains constant.

#### Parabolic Motion of Projectiles

Let us consider a ball projected at an angle  $\theta$  with respect to the horizontal x-axis with the initial velocity  $u$  as shown below:



The point **O** is called the **point of projection**;  $\theta$  is the **angle of projection** and **OB = Horizontal Range** or Simply Range. The total time taken by the particle from reaching O to B is called the **time of flight**.

For finding different parameters related to projectile motion, we can make use of differential **equations of motions**:

①	$v = u - gt$
②	$s = ut - \frac{1}{2}gt^2$
③	$v^2 = u^2 - 2gs$

**u = Initial velocity | g = Acceleration due to gravity**  
**t = Time | s = Displacement | v = Final velocity**

### Projectile motion problems:

#### Problem (1):

A person kicks a ball with an initial velocity of 15m/s at an angle of  $37^\circ$  above the horizontal (neglect the air resistance). Find

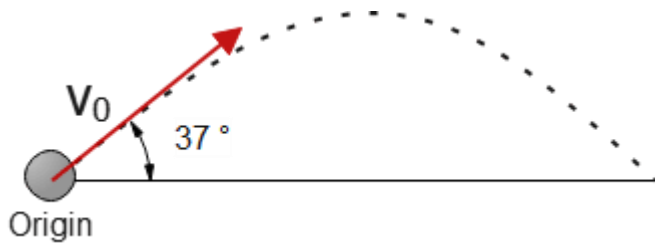
- (a) the total time the ball is in the air.
- (b) The horizontal distance traveled by the ball

#### Solution:

To solve any projectile motion problems, first of all, adopt a coordinate system and draw its projectile path, and put the initial and final positions and velocities on it.

By doing so, you will be able to solve the relevant projectile equations easily.

Hence, we choose the origin of the coordinate system to be at the throwing point  $X_0=0$ ,  $y_0=0$ ;



(a) Here, the time between throwing and striking the ground is wanted.

In effect, the projectiles have two independent motions, one is in the horizontal direction with uniform motion at a constant velocity, i.e.,  $a_x=0$ , and the other is in the vertical direction under the effect of gravity with  $a_y=-g$ .

The kinematic equations that describe the horizontal and vertical distances are as follows

$$x = x_0 + \underbrace{(v_0 \cos \theta)}_{v_{0x}} t$$

$$y = -\frac{1}{2}gt^2 + \underbrace{(v_0 \sin \theta)}_{v_{0y}} t + y_0$$

By substituting the coordinates of the initial and final points into the vertical equation, we can find the total time the ball is in the air.

Setting  $y = 0$  in the second equation (because the projectile lands at the same level of throwing point.), we have

$$y = -\frac{1}{2}gt^2 + (v_0 \sin \theta)t + y_0$$

$$0 = -\frac{1}{2}(9.8)t^2 + (15) \sin 37^\circ t + 0$$

By rearranging the above expression, we can get two solutions for  $t$ :

$$t_1 = 0$$

$$t_2 = \frac{2 \times 15 \sin 37^\circ}{9.8} = 1.84 \text{ s}$$

The first time is for the starting moment and the second is the total time the ball was in the air.

**(b)** As mentioned above, the projectile motion is made up of two independent motions with different positions, velocities, and accelerations which two distinct kinematic equations describe those motions.

Between any two desired points in the projectile path, the time needed to move horizontally to reach a specific point is the same time needed to fall vertically to that point.

This is an important observation in solving projectile motion problems.

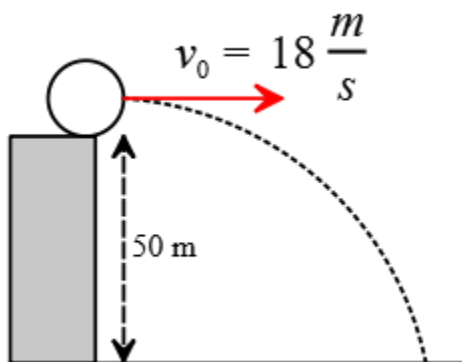
Therefore, time is the only common quantity in the horizontal and vertical motions of a projectile. In this problem, the time obtained in part (a) can be substituted in the horizontal kinematic equation, to find the distance travelled as below

$$\begin{aligned}x &= x_0 + (v_0 \cos \theta)t \\ &= 0 + (15) \cos 37^\circ (1.84) \\ &= 22.08 \text{ m}\end{aligned}$$

**Problem (2):**

A person standing on the edge of a 50 m-high cliff throws a stone horizontally with a speed of 18m/s.

- (a) what is the initial position of the stone?
- (b) What are the components of the initial velocity?
- (c) What are the  $x$ - and  $y$ -components of the velocity of the stone at any arbitrary time  $t$ ?
- (d) How long will it take the stone to strike the bottom of the cliff?
- (e) With what angle and speed do the stone strike the ground below the cliff?

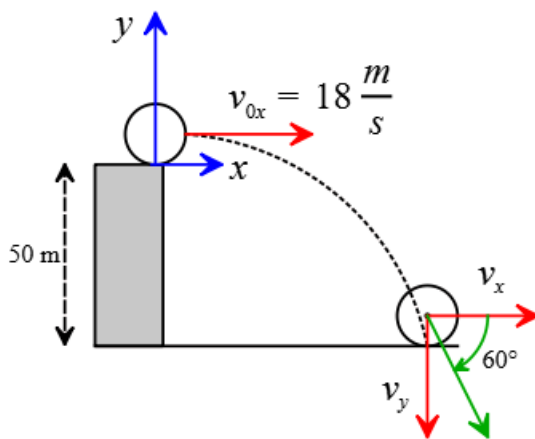


**Solution:**

As mentioned repeatedly, as a first step to solve a projectile motion problem, choose a relevant coordinate system.

**(a)** Usually, we place the origin of the coordinate system at the point where the projectile is thrown. In this case, the coordinate of the initial position is  $x_0=0, y_0=0$ .

If we had chosen the coordinate at the base of the cliff and placed the origin at that point, the position of the initial point would have been  $x_0 = 0, y_0 = 50 \text{ m}$ .



**(b)** The stone is thrown horizontally,  $\theta = 0$ , so there have not any vertical velocity component. Consequently, their initial speed components are  $v_{0x} = 18 \text{ m/s}$  and  $v_{0y} = 0$ .

**(c)** Recall that the components of the velocity of a projectile vary with time according to the following formula. Substitute the given values into it, gives

$$v_x = v_{0x} = 18 \text{ m/s}$$

$$\begin{aligned} v_y &= v_{0y} - gt \\ &= -9.8t \end{aligned}$$

**(d)** If we take the top of the cliff as the origin of our coordinate system,  $x_0 = y_0 = 0$ , the stone strikes the ground 50 m below our chosen origin, so the coordinate of that point would be  $x_0 = ?$ ,  $y = -50$  m.

Use the vertical displacement kinematic equation below, substitute the numerical values into that, and solve for time  $t$  get

$$y - y_0 = -\frac{1}{2}gt^2 + v_{0y}t$$

$$-50 - 0 = -\frac{1}{2}(9.8)t^2 - 0$$

$$\Rightarrow \boxed{t = 3.2 \text{ s}}$$

**(e)** First, find the velocity components just before the stone hit the ground. We know that it takes about 3.2 s for the stone to reach the bottom of the cliff. Thus, put this time value into the formulas of velocity components at any time  $t$ .

$$v_x = v_{0x} = 18 \text{ m/s}$$

$$\begin{aligned} v_y &= v_{0y} - gt \\ &= -9.8 \times 3.2 \\ &= -31.36 \text{ m/s} \end{aligned}$$

Notice that the negative sign here indicates that the vertical velocity component just before hit the ground points downward, as expected.

Recall from the section of vector practice problems that having the components of a vector, its magnitude is simply found using the Pythagorean Theorem as follows

$$\begin{aligned}
 v &= \sqrt{v_x^2 + v_y^2} \\
 &= \sqrt{18^2 + (-31.36)^2} \\
 &= \boxed{36.15 \text{ m/s}}
 \end{aligned}$$

The angle of impact with the ground, having the velocity components, is also obtained as below

$$\begin{aligned}
 \tan \theta &= \frac{v_y}{v_x} \\
 &= \frac{-31.36}{18} \\
 &= -1.74
 \end{aligned}$$

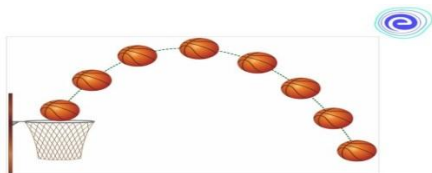
Taking the inverse tangent of both sides, gives

$$\theta = \tan^{-1}(-1.74) = \boxed{-60.1^\circ}$$

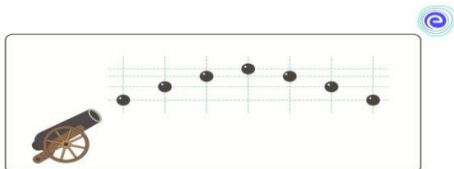
Therefore, the stone hit the ground at an angle of about  $60^\circ$  with a speed of  $36.15 \text{ m/s}$ . The negative indicates that the angle is below horizontal, which is to be expected.

### Applications of projectile

1. A basketball player should know the angle at which to throw the ball in order to put it in the basket to score.



2. Canon should know the angle at which it should fire to hit its target.



3. Football
4. Golf Balls
5. Rockets and Missiles

### 3.3.3. Equations of projectile motion

**The velocity of projection:** It is the velocity with which the body is projected. It is denoted by  $u$

**The angle of projection:** It is the angle with the horizontal at which the body is projected. It is denoted by  $\theta$ .

$$\begin{cases} v_{x,0} = v_0 \cos(\theta) \\ v_{y,0} = v_0 \sin(\theta) \end{cases}$$
$$\begin{cases} v_{x,t+dt} = v_{x,t} \\ v_{y,t+dt} = v_{y,t} - \frac{1}{2} g dt \end{cases}$$
$$\begin{cases} x_{t+dt} = x_t + v_{x,t} dt \\ y_{t+dt} = y_t + v_{y,t} dt \end{cases}$$



#### Theoretical learning Activity

In groups of four, discuss on the following problems

1. What is a projectile?
2. What is a trajectory?
3. Define time of flight.



#### Points to Remember

**A projectile** is any object thrown into space upon which the only acting force is gravity.

**In a Projectile Motion, there are two simultaneous independent rectilinear motions:**

1. **Along the x-axis:** uniform velocity, responsible for the **horizontal** (forward) **motion** of the particle.
2. **Along the y-axis:** uniform acceleration, responsible for the **vertical** (downwards) **motion** of the particle.



## Learning outcome 3 Formative Assessment

### Written assessment

1. Draw Cartesian planes and locate the vectors provided. Draw a line joining the two points and indicate the arrow showing the direction from the first to the last point.
  - A. A (2,2) and B (3,1)
  - B. P (5,3) and Q (2,2)
  - C. P (3,1) and Q (2, -3)
  - D. A (3,4) and B (4,10)
  - E. A (2, -3) and B (6,7)
  - F. A (5,1) and B (2, -3)
2. Without drawing, find the resultant vectors for the following points.
  - a) A (2,0) and B (3, -11)
  - b) P (5,1) and Q (2,4)
  - c) P (-6,1) and Q (6, -3)
  - d) A (3,2) and B (4, -5)

### Solution

a)

To find the resultant vector  $\vec{AB}$  from points A(2,0) to B(3,-11), you can use the following formula:

$$\vec{AB} = \vec{B} - \vec{A}$$

Given that A has coordinates (2,0) and B has coordinates (3,-11), subtract the corresponding components:

$$\vec{AB} = (3 - 2, (-11) - 0)$$

Simplify:

$$\vec{AB} = (1, -11)$$

So, the resultant vector  $\vec{AB}$  is (1, -11).

b)

To find the resultant vector  $\vec{PQ}$  from points P(5,1) to Q(2,4), you can use the following formula:

$$\vec{PQ} = \vec{Q} - \vec{P}$$

Given that P has coordinates (5,1) and Q has coordinates (2,4), subtract the corresponding components:

$$\vec{PQ} = (2 - 5, 4 - 1)$$

Simplify:

$$\vec{PQ} = (-3, 3)$$

So, the resultant vector  $\vec{PQ}$  is (-3, 3).

3.

If the position vector of the particle is given by  $\vec{r} = 3t^2\hat{i} + 5t\hat{j} + 4\hat{k}$ , Find the

- a. The velocity of the particle at  $t = 3$  s
- b. Speed of the particle at  $t = 3$  s
- c. acceleration of the particle at time  $t = 3$  s

4. a) differentiate vector and scalar quantities

b) Give two example for examples for each

c. Find the addition of the following vectors



i) Using tail-to-tip method

ii) Using parallelogram method

### Solution

a) Vectors are those quantities with both magnitude and direction **while** scalar have only magnitude

b) Give two example for examples for each

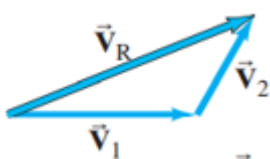
Vectors: force, velocity, displacement, acceleration...

Scalar: speed, distance, time, mass ....

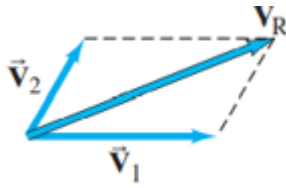
c) Find the addition of the following vectors



a.tail-to tip method



b. parallelogram method



5. A Thrown Baseball The position of a thrown baseball is given by:

$$\vec{r} = [1.5 \text{ m} + (12 \text{ m/s})t]\hat{x} + [(16 \text{ m/s})t - (4.9 \text{ m/s}^2)t^2]\hat{y}$$

(a) Find the velocity as a function of time.

(b) Find the acceleration as a function of time.

**Solution**

(a) the velocity as a function of time.

$$x = 1.5 \text{ m} + (12 \text{ m/s})t; \quad y = (16 \text{ m/s})t - (4.9 \text{ m/s}^2)t^2$$

$$v_x = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = 12 \text{ m/s}; \quad v_y = \lim_{\Delta t \rightarrow 0} \frac{\Delta y}{\Delta t} = (16 \text{ m/s}) - 2(4.9 \text{ m/s}^2)t$$

$$\vec{v} = (12 \text{ m/s})\hat{x} + [(16 \text{ m/s}) - (9.8 \text{ m/s}^2)t]\hat{y};$$

(b) the acceleration as a function of time.

$$a_x = \lim_{\Delta t \rightarrow 0} \frac{\Delta v_x}{\Delta t} = 0; \quad a_y = \lim_{\Delta t \rightarrow 0} \frac{\Delta v_y}{\Delta t} = -9.8 \text{ m/s}^2$$

$$\vec{a} = (-9.8 \text{ m/s}^2)\hat{y}$$

6. A sailboat has coordinates (130 m, 205 m) at zero second. Two minutes later its position is (110 m, 218 m).

- (a) Find the average velocity vector
- (b) Find the magnitude of average velocity vector
- (c) Find the tangent of angle
- (d) True or false
  - i) Velocity is a scalar vector
  - ii) Acceleration is a vector quantity
  - ii) Volume is a vector quantity

**Solution**

$$\vec{v}_{av} = v_{xav}\hat{x} + v_{yav}\hat{y}$$
$$v_{xav} = \frac{\Delta x}{\Delta t} = \frac{110 \text{ m} - 130 \text{ m}}{120 \text{ s}} = -0.167 \text{ m/s}$$
$$v_{yav} = \frac{\Delta y}{\Delta t} = \frac{218 \text{ m} - 205 \text{ m}}{120 \text{ s}} = 0.108 \text{ m/s}$$
$$\vec{v}_{av} = (-0.167 \text{ m/s})\hat{x} + (0.108 \text{ m/s})\hat{y}$$
$$v_{av} = \sqrt{(-0.167 \text{ m/s})^2 + (0.108 \text{ m/s})^2} = 0.199 \text{ m/s}$$

**c) tangent of angle**

$$\tan(\emptyset) = \frac{v_y}{v_x} = \frac{-0.108}{0.167}$$

$$\emptyset = -32.89^\circ$$

d) True or false

i) False

ii) True

iii) False

## Practical assessment

### Practical Activity: Investigating Projectile Motion

**Objective:** To explore and analyze the key aspects of projectile motion, including the range, time of flight, and the effect of launch angle on the trajectory.

#### Materials Needed:

7. Balls or small projectiles
8. Measuring tape
9. Stopwatch or timer
10. Open outdoor space or a gymnasium with sufficient clearance
11. Graph paper and markers (optional, for data visualization)

#### Procedure:

##### Setting Up the Experiment:

- a. Choose an open area with ample space, ensuring safety and clearance for the projectiles.
- b. Mark a horizontal line on the ground to serve as the launch point.

##### Measuring Launch Angle:

- a. Use a protractor to measure and set the launch angle. Start with a relatively low angle (e.g., 30 degrees) and plan to test multiple angles.

Launching the Projectiles: a. Stand at the launch point and launch the projectiles at the chosen angle.

- b. Record the launch angle for each trial.

##### Measuring Range:

- a. Measure the horizontal distance travelled by the projectile (range) for each launch.
- b. Repeat the experiment for different launch angles.

##### Timing the Flight:

- a. Use a stopwatch to measure the time of flight for each projectile.
- b. Ensure that the timing starts when the projectile is launched and stops when it hits the ground.

**Data Collection:**

- a. Record the launch angles, ranges, and times of flight for each trial in a table.
- b. Optionally, use graph paper to create a visual representation of the projectile trajectories.

**Analysis:**

- a. Calculate the average range and average time of flight for each launch angle.
- b. Analyze the relationship between launch angle and range.

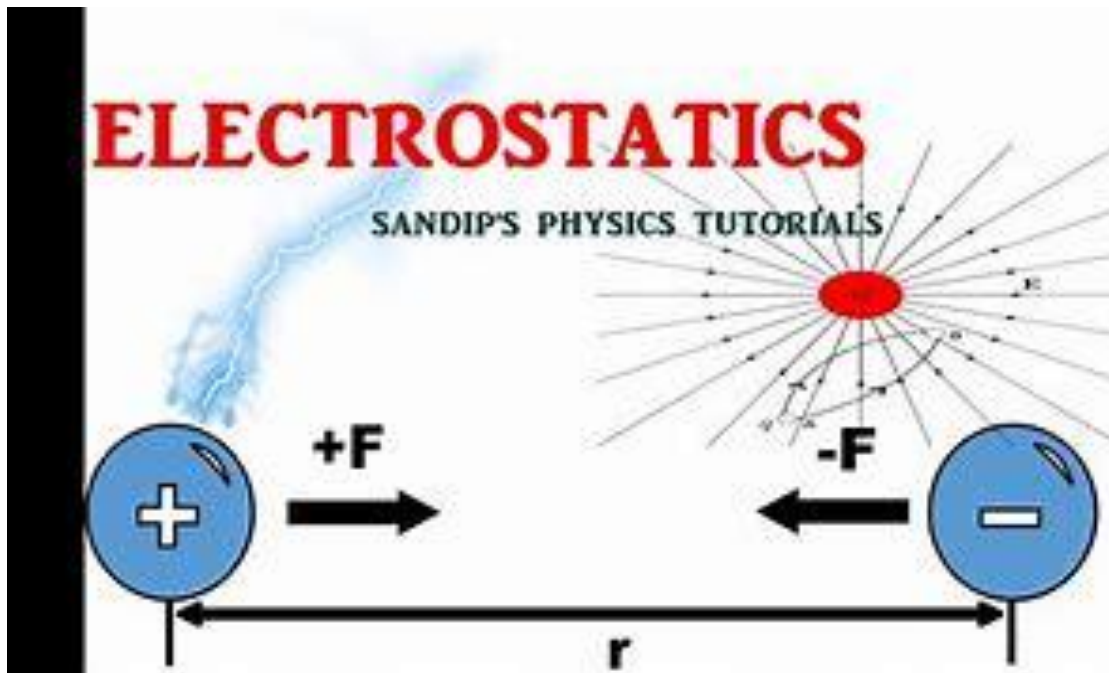
**Effect of Launch Angle:**

- a. Discuss the impact of launch angle on the trajectory. How does it affect the range and time of flight?
- b. Explore the concept of the optimum launch angle for maximum range.

**Comparison and Conclusion:**

- a. Compare the results for different launch angles and discuss any trends or patterns observed.
- b. Summarize the key findings and draw conclusions about the factors influencing projectile motion.

## Learning Outcome 4: Demonstrate Electrostatic Phenomena



### Learning outcome 4. Demonstrate electrostatic phenomena

#### Indicative contents:

- 4.1. Description of electrostatic charges and their conservation
- 4.2. Determination of the electrostatic fields
- 4.3. Demonstration of effects of electric field on charged particles



**Duration: 8 hours**



#### Learning outcome 4 objectives:

By the end of the learning outcome, the trainees will be able to:

1. Describe correctly Electrostatic charges and their conservation in line with laws of static charges on Coulomb's law and Gauss's law
2. Determine effectively Electrostatic fields based on Coulomb's law
3. Demonstrate clearly Effects of electric field on charged particles based on fundamental laws of static charges



## Resources

Equipment	Tools	Materials
- PPE, whiteboard and chalkboard, computer, projector, textbooks	- Scientific calculator, meter ruler, compass	- Chalks, markers



## Advance preparation:

Prepare in advance a video or any simulation on electrostatics.



## 4.1: Description of electrostatic charges and their conservation

### 4.1.1. Electric charges

**Electrostatics** is a branch of Physics that deals with the phenomena and properties of stationary electric charges.

The **elementary charge**, usually denoted by  $e$  is the electric charge carried by a single proton or, equivalently, the magnitude of the negative electric charge carried by a single electron, which has charge  $-1 e$ . This elementary charge is a fundamental physical constant.

In the SI system of units, the value of the elementary charge is exactly defined as  $e = 1.602176634 \times 10^{-19}$  coulombs, or 160.2176634 C.

A **point charge** is a hypothetical charge located at a single point in space.

### Static Electricity

A nylon garment often crackles when it is taken off. We say it has become charged with static electricity. The crackles are caused by tiny electric sparks which can be seen in the dark. Pens and combs made of certain plastics become charged when rubbed on the sleeve and can attract scraps of paper. Those materials are electrified, possess an electric charge or are electrically charged.

There are two kinds of charges in nature; **negative** and **positive charges**.

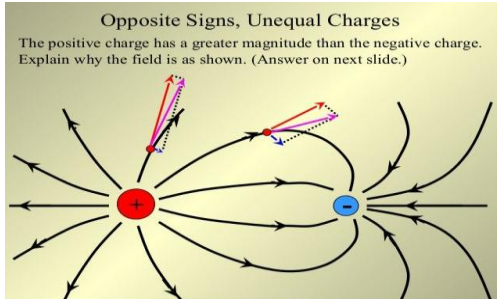
### Laws of electrostatic charges

Like charges (+ and + or – and –) **repel** while unlike charges (+ and –) **attract**.

The net amount of electric charge produced in any process is zero. This is known as **the Law of Conservation of Electric Charge**. If one object or one region of space acquires a positive charge then an equal amount of negative charge will be found in neighbouring areas or objects.

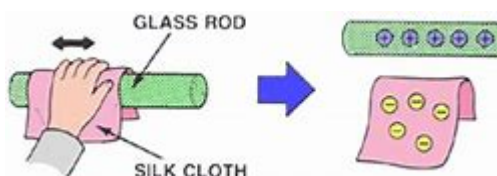
## Sign and magnitude of electric charges

**Electric charge** is the physical property of matter that causes it to experience a force when placed in an electromagnetic field.



### 4.1.2. Electrification (charging) methods

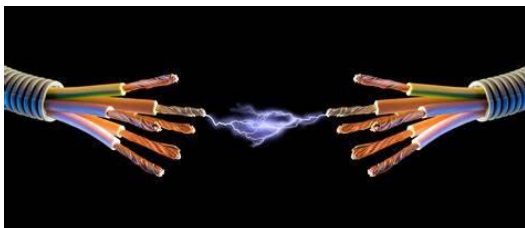
#### a. Electrification by rubbing



When two bodies are rubbed together there will be a **transfer of electrons from one body to the other body**.

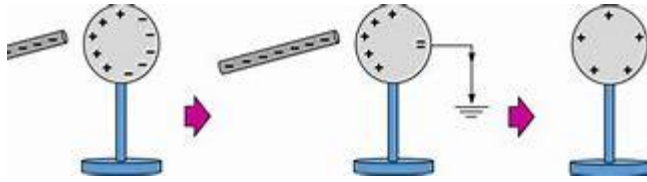
#### b. Electrification by contact

**Contact electrification** can occur whenever two surfaces touch.



#### c. Electrification by induction

According to Merriam-Webster, induction is "the process by which an electrical conductor becomes electrified when near a charged body, by which a magnetisable body becomes magnetized when in a magnetic field or in the magnetic flux set up by a magneto motive force.



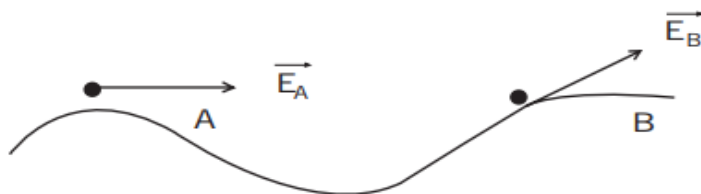
### 4.1.3. Electrostatic field

#### The Concept of Electric Field

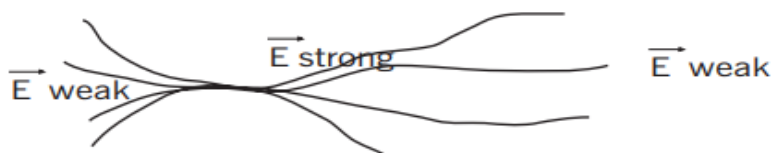
When a small charged particle is located in the area surrounding a charged object, the charged particle experiences a force in accordance with Coulomb's Law. The space around the charged object where force is exerted on the charged particle is called **an electric field or electrostatic field**. Theoretically, an electric field due to charge extends to infinity but its effect practically dies away very quickly as the distance from the charge increases.

#### Electric field lines

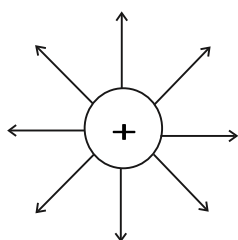
Electric field lines (or **line of force** in an electric field) are an imaginary line drawn through an area or place. The line is drawn in such a way that the direction of its tangent is the same as the direction of the electric field at that point.



The spacing of field lines gives a general idea of the magnitude of  $\vec{E}$  at each point. Where  $\vec{E}$  is strong the electric field lines are drawn closely together. Where  $\vec{E}$  is weaker, the electric field lines are further apart.

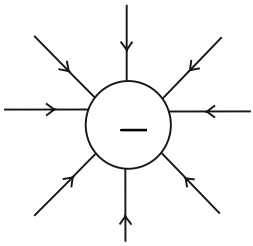


a) Electric field lines produced by a single positive point charge.



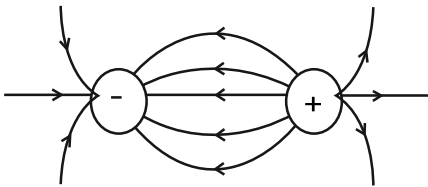
The electric field lines always point away from positive charge

b) Electric field lines produced by a single negative point charge.



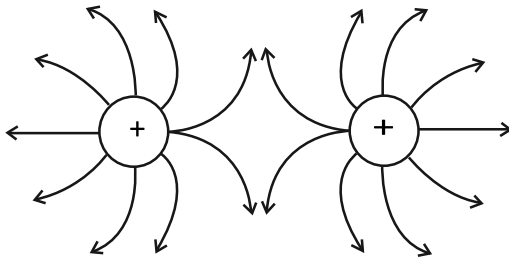
The electric field lines always point towards a negative charge.

c) Electric field lines produced by two equal and opposite point charges

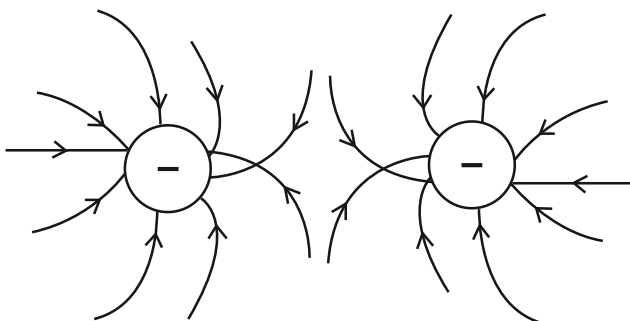


**Note: The number of lines leaving the positive charge equals the number entering the negative charge.**

d) Electric field lines for two equal and positive charges.



e) Electric field lines for two equal and negative charges.

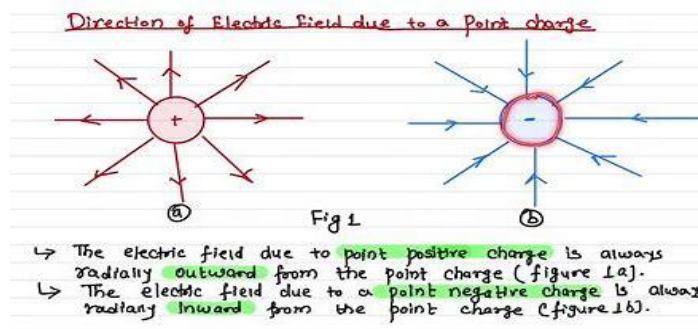


## Electric field at a point charge

An electric field is also described as the electric force per unit charge.

The formula of electric field is given as;

$$E = F / Q.$$



## Electric field between two charges

The electric field (E) between two charges is a vector field that describes the force experienced by a positive test charge placed in that region. The electric field is generated by the charges and provides information about the force that a positive charge would experience at any point in the field. The electric field (E) is defined as the force (F) per unit positive charge (Q<sub>0</sub>)

$$E = \frac{F}{Q_0}$$

The electric field due to a point charge (Q) at a distance (r) from the charge is given by

Coulomb's Law:  $E = \frac{kQ}{r}$

Where:

E is the electric field,

k is Coulomb's constant ( $\approx 8.99 \times 10^9 \text{ N m}^2/\text{C}^2$ ),

Q is the charge creating the electric field,

r is the distance from the charge to the point where the field is measured.

### Example

Suppose we have a point charge  $Q=5\mu\text{C}$  (micro coulombs). We want to find the electric field at a point  $P$  located  $3\text{m}$  away from the charge.

Given:

- Charge  $Q = 5 \mu\text{C}$
- Distance from the charge to point  $P$  ( $r$ ) =  $3 \text{ m}$
- Coulomb's constant  $k \approx 8.99 \times 10^9 \text{ N m}^2/\text{C}^2$

We can use Coulomb's Law to find the electric field ( $E$ ) at point  $P$ :

$$E = \frac{k \cdot Q}{r^2}$$

Substitute the known values:

$$E = \frac{(8.99 \times 10^9 \text{ N m}^2/\text{C}^2) \cdot (5 \times 10^{-6} \text{ C})}{(3 \text{ m})^2}$$

$$E \approx \frac{(8.99 \times 10^9 \text{ N m}^2/\text{C}^2) \cdot (5 \times 10^{-6} \text{ C})}{9 \text{ m}^2}$$

$$E \approx \frac{44.95 \text{ N m/C}}{9 \text{ m}^2}$$

$$E \approx 4.995 \text{ N/C}$$



### Theoretical learning Activity

**In group of four, trainees discuss the following:**

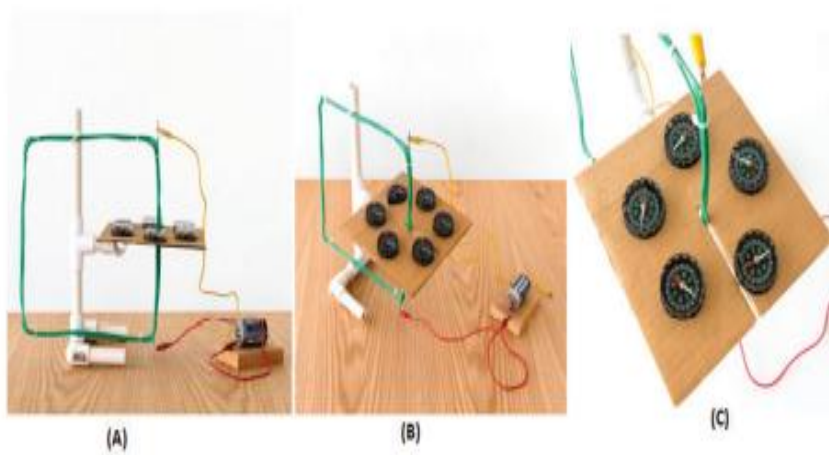
1. A particle has a charge of  $-8.00\text{nC}$ . Find the magnitude and direction of the electric field due to this particle at a point  $0.5\text{m}$  directly above it.
2. What is the electric charge?
3. What is the value of the elementary charge?
4. What is electrostatic?
5. What are the methods of electrification?
6. Define electric field lines



## Practical learning Activity

Investigation of electric field Materials:

- One battery cell (1.5V)
- A conducting wire
- 5 magnetic needles
- A slotted cardboard Procedure:
- Arrange the materials as shown in Fig. below



*Figure 27: Electric field effect on the magnetic needles*

- Remove the battery and note the changes on needles.
- Reconnect the battery and note the changes on needles.

### Question:

What is the main cause of the directions change when the battery is connected?



## Points to Remember

- **Electrostatics** is a branch of Physics that deals with the phenomena and properties of stationary electric charges
- Like charges (+ and + or – and -) **repel** while unlike charges (+ and -) **attract**.
- Electric field lines (or **line of force** in an electric field) are an imaginary line drawn through an area or place.
- **Electrification (charging) methods**
  - d. Electrification by rubbing
  - e. Electrification by contact
  - f. Electrification by induction



## 4.2: Determination of the electrostatic fields

### 4.2.1. Coulomb's law of electrostatic charges

**Permittivity**, also called electric permittivity, is a constant of proportionality that exists between electric displacement and electric field intensity. This constant is equal to approximately  $8.85 \times 10^{-12}$  farad per meter (F/m) in free space (a vacuum).

**Coulomb's law states** that the magnitude of the electrostatic force between two-point charges is directly proportional to the product of the magnitudes of the charges and inversely proportional to the square of the distance between them.

### Mathematical treatment of coulombs of electrostatic charges

$$F = k \cdot \frac{q_1 \cdot q_2}{r^2}$$

where:

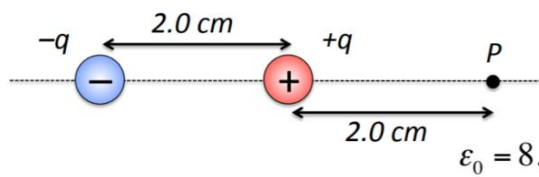
- $F$  is the electrostatic force between the two charges,
- $k$  is Coulomb's constant, approximately  $8.99 \times 10^9 \text{ N m}^2/\text{C}^2$ ,
- $q_1$  and  $q_2$  are the magnitudes of the two point charges,
- $r$  is the separation distance between the charges.

### 4.2.2. Electric field intensity and potential

The **space around an electric charge** in which its influence can be felt is known as the electric field. The electric field intensity at a point is the **force experienced by a unit positive charge placed** at that point.

1. Electric Field Intensity is a vector quantity.
2. It is denoted by 'E'.
3. Formula: Electric Field = F/q.
4. Unit of E is  $\text{NC}^{-1}$  or  $\text{Vm}^{-1}$ .

An electric dipole: if the electric field strength at point P is  $E = 6068 \text{ N/C}$ , what is the charge  $q$ ?



For single point charge:

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$$

A. 0.36 nC

C. 0.22 nC

Answer C is adding the magnitudes of  $E_+$  and  $E_-$  together

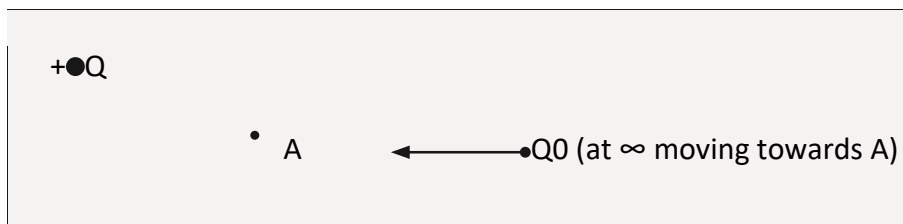
After looking into it, I'm not sure what misconception answer B relates to

B. 0.27 nC

D. 0.13 nC

### Electric potential

Every charge has an electric force, which extends theoretically up to infinity. Let us consider an isolated charge  $+Q$  fixed in space:



If a test charge  $Q_0$  is placed at infinity, the force on it due to charge  $+Q$  is zero.

$$F = 9 \times 10^9 \frac{QQ_0}{d^2} \text{ as } d \rightarrow \infty F \rightarrow 0$$

If the test charge  $Q_0$  at infinity is moved towards  $+Q$  a force of repulsion acts on it and hence work is required to be done to bring it to a point like A. The work done by the electric force does not depend on the path taken by charge  $Q_0$ , it is only dependent on the initial and final position, i.e. the electric force is conserved and the work done can be expressed in terms of potential energy,  $U$ .

The work done in bringing  $Q_0$  from infinity to A is given by:

$$W = F \cdot d = \frac{QQ_0}{4\pi\epsilon d^2} \cdot d$$

$$W = \frac{QQ_0}{4\pi\epsilon d}$$

Then:

$$W_{\infty \rightarrow A} = U_B - U_A = -(U_A - U_{\infty})$$

$$= -\Delta u \quad (\Delta u = \text{change in energy})$$

Where:

$$U_{\infty} = \frac{QQ_0}{4\pi\epsilon d_{\infty}}$$

Is the potential energy of the test charge  $Q_0$  at  $d = \infty$

$$U_A = \frac{QQ_0}{4\pi\epsilon d_A}$$

Is the potential energy of the test charge  $Q_0$  at  $d = A$

Hence, the electric potential energy  $U$  of a test charge  $Q_0$  placed at distance  $d$  from the charge  $+Q$  is given by

$$U = \frac{QQ_0}{4\pi\epsilon d}$$

The electric potential  $V$  at a point distance  $d$  from the charge  $Q$  is the electric potential energy  $U$  per unit charge associated with a test charge  $Q_0$  placed at that point:

$$V = \frac{U}{Q_0} \quad \text{Then}$$

$$V = \frac{QQ_0}{4\pi\epsilon Q_0 d}$$

$$V = \frac{Q}{4\pi\epsilon d}$$

Hence, electric potential at a point in an electric field is the amount of work done in bringing a unit of positive charge from infinity to that point, i.e.

$$V = \frac{\text{Work}}{\text{Charge}} = \frac{W}{Q}$$

Where  $W$  is the work done to bring a charge of Coulomb from infinity to the point of consideration.

**Unit: The SI unit of electric potential is volt (V) and may be defined as:**

*"The potential at a point in an electric field is 1 volt if 1 joule of work is done in bringing a unit of positive charge from infinity to that point against the electric field."*

#### 4.2.3. Effect of an electrostatic field on a moving charge

The charge deflection is independent of the mass and the charge, so this experiment cannot be used to measure  $e/m$ . The reason that it is independent of these values is that, if the charge increases, then the accelerating force increases by the same amount in the electron gun and between the deflection plates. A similar argument applies to any changes of mass.

**Charge acceleration** it is easy to see why acceleration is necessary: a charge at rest produces only a static electric field. A charge moving at constant velocity, as in a current, produces a static magnetic field. To have the fields change with time, as in an electromagnetic wave, the charges must not be at rest or moving at constant velocity.



### Theoretical learning Activity

In groups of four, trainees discuss the following:

1. What is permittivity?
2. What is the statement of coulomb's law?
3. Define electric field



### Points to Remember

- **Permittivity**, also called electric permittivity, is a constant of proportionality that exists between electric displacement and electric field intensity. This constant is equal to approximately  **$8.85 \times 10^{-12}$  farad per meter (F/m)** in free space (a vacuum).
- The **space around an electric charge** in which its influence can be felt is known as the electric field.



## 4.3 : Demonstration of effects of electric field on charged particles

### 4.3.1. CAPACITORS

Fig below shows an electric circuit of d.c and a.c sources

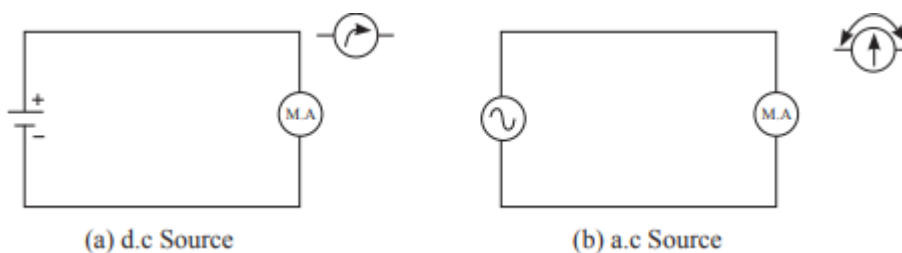
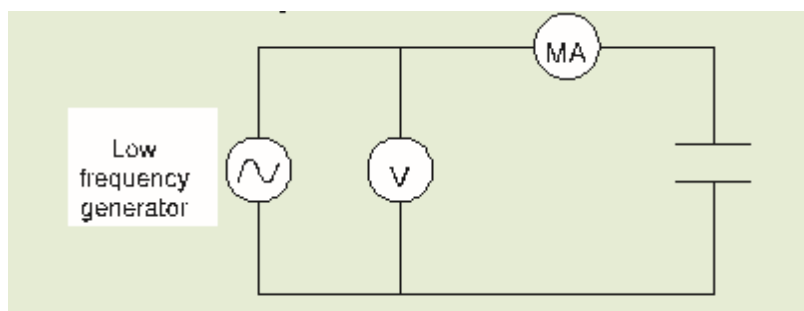


Fig. : The d.c and a.c source

**Capacitor** is an **electronic component** capable of storing electrical energy in electric field, especially one consisting of two conductors separated by dielectric.

	Capacitor (C)	It acts as short circuit with a.c and open with d.c
--	---------------	---

#### A single capacitor connected in series to an a.c source



**Capacitance (C)** is a fundamental property of a capacitor that quantifies its ability to store electric charge per unit voltage. It is defined as the ratio of the magnitude of the electric charge (Q) stored on one plate of the capacitor to the voltage (V) across the plates. Mathematically, capacitance is expressed by the formula:

$$C = \frac{Q}{V}$$

Where:

C is the capacitance,

Q is the magnitude of the electric charge stored on one of the capacitor plates,

V is the voltage across the plates.

The unit of capacitance in the International System of Units (SI) is the farad (F).

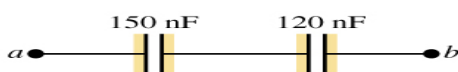
**A parallel plate capacitor** is a device used in electrical circuits to store electric charge. It consists of two parallel conducting plates separated by a dielectric material (insulator). The capacitor stores electric energy in an electric field formed between the plates.

### Effective capacitance for capacitor network

#### a) in series

Capacitors can be connected in series: The equivalent capacitance for series-connected capacitors can be calculated as

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$$

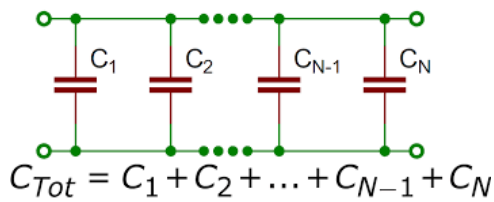


#### b) in parallel

When simplified, is the expression for the equivalent capacitance of the parallel network of three capacitors:

$$C_n = C_1 + C_2 + \dots + C_n$$

This expression is easily generalized to any number of capacitors connected in parallel in the network.



### 4.3.2. Electrostatic energy stored by a capacitor

**The energy stored in a capacitor** is electrostatic potential energy and is thus related to the charge  $Q$  and voltage  $V$  between the capacitor plates. A charged capacitor stores energy in the electrical field between its plates. As the capacitor is being charged, the electrical field builds up. When a charged capacitor is disconnected from a battery, its energy remains in the field in the space between its plates.

#### How to calculate the energy stored in a capacitor

Capacitor energy formula

$$E = \frac{1}{2} CV^2$$

Using the general formula for capacitance,

$$C = \frac{Q}{V}$$

We can rewrite the capacity energy equation in two other analogous forms:

$$E = \frac{1}{2} QV$$

#### Example

How much energy can be stored in a capacitor with capacity  $C = 300 \mu\text{F}$ , when we connect it to a voltage source of  $V = 20 \text{ V}$ ?

To make our life easier, use scientific notation for the capacitance:  $C = 3 \cdot 10^{-4} \text{ F}$

Following the capacity energy formula, the outcome is evaluated as:

$$E = 1/2 * 3 \cdot 10^{-4} \text{ F} * (20 \text{ V})^2 = 6 \cdot 10^{-2} \text{ J}$$

The energy stored in the capacitor can also be written as 0.06 J or 60 mJ

### 4.3.3. Examples of electrostatic phenomena

#### 1. Electrostatic discharge (ESD)

It is the release of static electricity when two objects come into contact

E.g. a balloon rubbed against one's hair.

#### 2. Lightning arrestors

Arrestors are typically installed near critical appliances or points of entry, such as an electrical panel or near a generator. When potentially dangerous lightning strikes, the arrestor activates and diverts the lightning to ground where it will disperse harmlessly

#### 3. Paint spraying

A positively charged electron within the spray nozzle charges the paint particles. Because these particles all have a positive charge, they repel each other and break apart, resulting in a fine mist coat evenly.

#### 4. Photocopier machines

Negatively charged powder spread over the surface adheres through electrostatic attraction to the positively charged image areas.

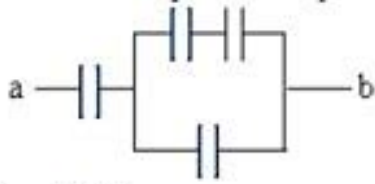


### Theoretical learning Activity

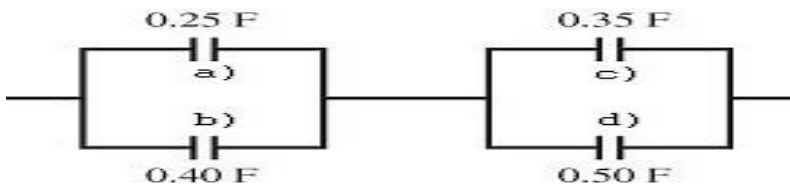
**In group of four, trainees discuss the following:**

1. What is capacitor and show its symbol?
2. Draw graphs which show the variation of capacitive reactance, against current, frequency of a.c source and capacitance.

What is the equivalent capacitance between points a and b? All capacitors are  $1.0 \mu\text{F}$ .



- a.  $4.0 \mu\text{F}$
- b.  $1.7 \mu\text{F}$
- c.  $0.60 \mu\text{F}$
- d.  $0.25 \mu\text{F}$



- 3. What is the energy stored in a capacitor?
- 4. Examples of electrostatic phenomena

## **lc** Points to Remember

- **Capacitor** is an electronic component capable of storing electrical energy in electric field, especially one consisting of two conductors separated by dielectric.

	Capacitor (C)	It acts as short circuit with a.c and open with d.c
--	---------------	---

- For capacitors in an a.c circuits, the instantaneous current is at its minimum or zero whenever the applied voltage is at its maximum and likewise the instantaneous value of the current is at its maximum or peak value when the applied voltage is at its minimum or zero.
- The energy stored in a capacitor is electrostatic potential energy and is thus related to the charge  $Q$  and voltage  $V$  between the capacitor plates.



## Learning outcome 4. Formative Assessment

### Written assessment

For questions 1 Select the most appropriate answer for the choices given.

1. as the size of the plates in a capacitor increases, all other factors being constant,

A. The value of  $X_C$  increases negatively

**B. The value of  $X_C$  decreases negatively.**

C. The value of  $X_C$  does not change.

D. We cannot say what happens to  $X_C$  without more data.

2. If the dielectric material between the plates of a capacitor is changed, all other things being equal,

A. The value of  $X_C$  increases negatively.

B. The value of  $X_C$  decreases negatively

C. The value of  $X_C$  does not change.

**D. We cannot say what happens to  $X_C$  without more data.**

3. As the frequency of a wave gets lower, all other things being equal, the value of  $X_C$  for capacitor

**A. Increases negatively.**

B. Decreases negatively.

C. Does not change.

D. Depends on the current.

4. What is the reactance of a 330-pF capacitor at 800 kHz?

A.  $-1.66 \Omega$     B.  $-0.00166 \Omega$     **C.  $-603 \Omega$**     D.  $-603 \text{ k}\Omega$

5. A capacitor has a reactance of  $-4.50 \Omega$  at 377 Hz. What is its capacitance?

- A.  $9.39 \mu\text{F}$     **B.  $93.9 \mu\text{F}$**     C.  $7.42 \mu\text{F}$     D.  $74.2 \mu\text{F}$

6. A  $47\text{-}\mu\text{F}$  capacitor has a reactance of  $-47 \Omega$ . What is the frequency?

- A.  $72 \text{ Hz}$**     B.  $7.2 \text{ MHz}$     C.  $0.000072 \text{ Hz}$     D.  $7.2 \text{ Hz}$

7. A capacitor has  $X_c = -8800 \Omega$  at  $f = 830 \text{ kHz}$ . What is C?

- A.  $2.18 \mu\text{F}$     **B.  $21.8 \text{ pF}$**     C.  $0.00218 \mu\text{F}$     D.  $2.18 \text{ pF}$

8. A capacitor has  $C = 166 \text{ pF}$  at  $f = 400 \text{ kHz}$ . What is  $X_c$ ?

- A.  $-2.4 \text{ k}\Omega$**     B.  $-2.4 \Omega$     C.  $-2.4 \times 10^{-6} \Omega$     D.  $-2.4 \text{ M}\Omega$

9. A capacitor has  $C = 4700 \mu\text{F}$  and  $X_c = -33 \Omega$ . What is f?

- A.  $1 \text{ Hz}$**     B.  $10 \text{ Hz}$     C.  $1 \text{ kHz}$     D.  $10 \text{ kHz}$

10. Briefly describe what happens when a capacitor is connected to an a.c source.

### Solution

- The capacitor alternately charges and discharges during each half-cycle of the AC signal.
- Capacitive reactance ( $X_C$ ) opposes the flow of AC current and depends on the frequency of the signal.
- There is a 90-degree phase shift between the voltage across the capacitor and the current.
- Higher frequencies result in lower capacitive reactance, allowing more current to pass through.

### **Practical assessment:**

**Practical Activity:** Electrification (Rubbing) Method on Hair Using a Pen

**Objective:** To explore the process of electrification through rubbing using everyday objects like a pen and human hair.

### **Materials Needed:**

1. Plastic or rubber pen
2. Human hair (clean and dry)
3. Small pieces of paper or lightweight objects (for testing attraction/repulsion)
4. Table or flat surface
5. Access to a dark room or area (optional)

**Procedure:**

**Introduction:**

- a. Start with a brief discussion on electrification and the concept of charging.
- b. Introduce the idea that certain materials can become charged through rubbing, and discuss everyday scenarios involving electrification.

**Selecting Materials:**

- a. Explain that in this activity, you will use a plastic or rubber pen and human hair.
- b. Discuss the characteristics of these materials and make predictions about which one might become charged through rubbing.

**Rubbing the Pen on Hair:**

- a. Hold the plastic or rubber pen and rub it vigorously against dry human hair for about 30 seconds.
- b. Discuss the sensation and any visible effects observed during rubbing.

**Testing Charged Pen:**

- a. Test the charged pen's ability to attract small pieces of paper or lightweight objects.
- b. Record observations and discuss the results.

**Charging in the Dark (Optional):**

- a. Repeat the rubbing method, but perform the experiment in a dark room or area.
- b. Observe any visible effects, such as a spark or glow, during charging.

**Exploration and Discussion:**

- a. Encourage participants to explore charging with different parts of the hair (e.g., near the scalp, at the tips).
- b. Discuss any variations in the charging process and results.

**Testing Repulsion (Optional):**

- a. Explore the concept of repulsion by charging the pen again and bringing it close to the charged hair.
- b. Observe any repulsive forces between the charged pen and the charged hair.

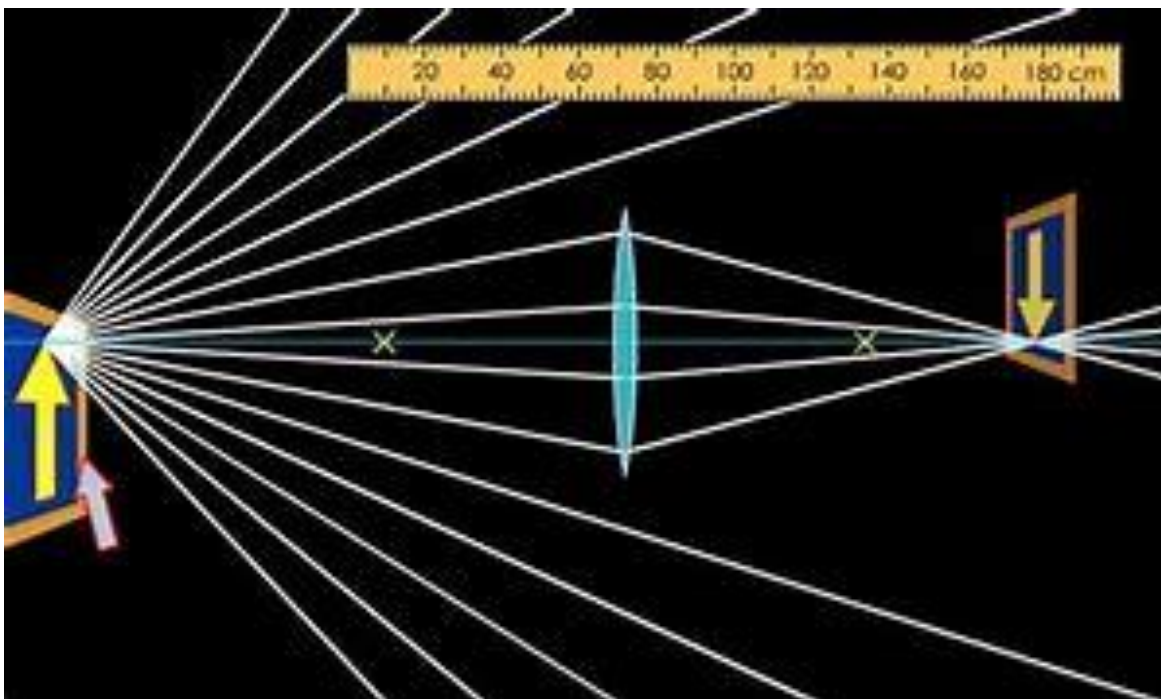
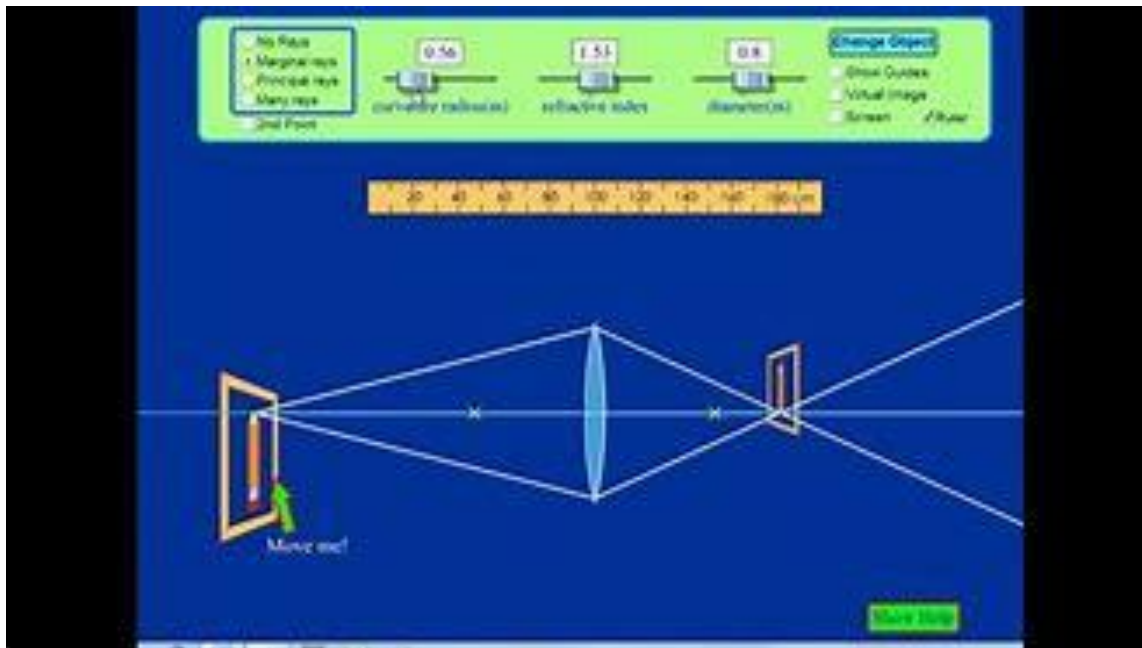
**Extension (Optional):**

- a. Experiment with different pens or materials to observe variations in electrification.
- b. Investigate the effect of humidity on the effectiveness of charging.

**Conclusion:**

- a. Summarize the key findings and concepts learned during the practical activity.
- b. Discuss the everyday relevance of electrification, such as the role of static electricity in hair.

## Learning outcome 5: Apply Geometric optics



## Learning outcome 5. Apply geometric optics

### Indicative contents:

- 4.1. Explanation of light
- 4.2. Application of light reflection on surfaces
- 4.3. Application of light refraction in different media



**Duration: 7 hours**



### Learning outcome 5 objectives:

**By the end of the learning outcome, the trainees will be able to:**

1. Explain properly the term light based on Fermat's principle
2. Apply effectively reflection of light on surfaces based on laws of reflection
3. Apply effectively refraction of light in different media based on Snell's law



### Resources

Equipment	Tools	Materials
- PPE, whiteboard, chalkboard, optical bench, optical slide, computer, projector, textbooks	- Scientific calculator, meter ruler, prism, thin lenses, compass	- Chalks, markers, candles, water



### Advance preparation:

- Prepare in advance Activities that illustrate reflection and refraction of light. The trainer can ask trainees to put a stick in water explain their observations.



## 5.1 : Description of properties of light in homogeneous medium

### 5.1.1. Key terms

**OPTICS:** The **Physics** of **light** and **vision**

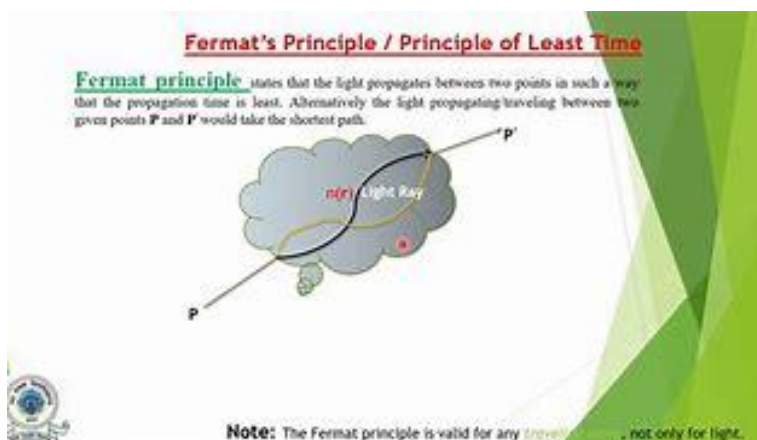
**Geometrical optics: or ray optics:** is a branch of optics that describes light propagation in terms of ray.

**What is the definition of light in Physics?**

**Light** is a form of electromagnetic radiation that enables the human eye to see or make things visible. It is also defined as radiation that is visible to the human eye. Light contains photons, which are minute packets of energy.

**Fermat's principle**, also known as the **principle of least time**, is the link between ray optics and wave optics. In its original "strong" form, Fermat's principle states that the path taken by a ray between two given points is the path that can be travelled in the least time.

**Fermat's principle**



## What is propagation of light?

Propagation of light refers to the manner in which an electromagnetic wave transfers its energy from one point to another. Three main processes generally occur when light passes between boundaries from one medium to another: Transmission.

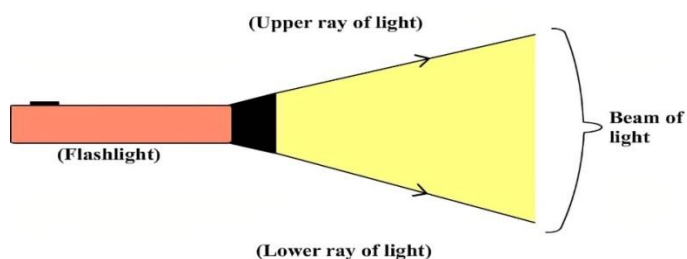


Figure 28: Beam of light

### ➤ RAY

A ray is an idealized narrow beam or a column of light. This is a very useful concept in geometrical optics. In geometrical optics, almost all of the calculations are done using light rays. An ideal light ray has zero width.

### ➤ BEAM

A beam is a narrow projection of a set of particles or waves. There are two main types of beams. Those are light (or electromagnetic) beams and particle beams. Beams are used in various fields and applications. Devices such as cathode ray tubes, particle accelerators, LASER devices use beams. Both types of beams can be considered as the same, since particles also have wave properties (and vice versa).

## Principle of reversibility of light

If a ray of light travel from medium 1 to medium 2 along a certain path, it retraces the path, when it passes from medium 2 to medium 1. Thus, the path of light is reversible.

### The properties of light are:

- ✓ Light is a form of energy.
- ✓ Light always travels along a straight line.
- ✓ Light does not need any medium for its propagation.
- ✓ Light can even travel through a vacuum.
- ✓ Light in different colours has different wavelengths and frequencies.



## Theoretical learning Activity

**In groups of four discuss the following:**

1. What is an optics?
2. Define what is light?
3. State Fermat's principle
4. What is the propagation of light?
5. Differentiate ray and beam?
6. What is the principle of reversibility of light



## Practical learning Activity

**Practical Activity:** Exploring Light with a Torch

**Objective:** To investigate and understand various properties of light using a torch and common materials.

**Materials Needed:**

1. Flashlight or torch
2. White wall or screen
3. Colored filters (optional)
4. Prisms (optional)
5. Mirror (small handheld or makeup mirror)
6. Various objects (transparent, translucent, and opaque)
7. Colored objects (e.g., colored paper, crayons)
8. Dark room or dimly lit area
9. Plain paper and pencils for recording observations

**Procedure:****Introduction:**

- a. Begin with a brief discussion on light and its properties.
- b. Introduce the practical activity's objectives and the materials to be used.

**Understanding Light Direction:**

- a. In a dark room, turn on the torch and observe how light travels in straight lines.
- b. Shine the torch in different directions and note how the light beam behaves.

**Reflection:**

- a. Shine the torch on a mirror and observe the reflection.
- b. Experiment with changing the angle of the mirror to see how it affects the direction of the reflected light.

**Refraction with Water:**

- a. Fill a transparent container with water and shine the torch through it.
- b. Observe the bending of light as it passes through the water.

**Dispersion with a Prism:**

- a. Shine the torch through a prism and observe the spectrum of colors produced.
- b. Explore how the angle of the prism affects the dispersion.

**Colored Filters:**

- a. Attach different colored filters to the front of the torch.
- b. Observe how the color of the light changes when passing through each filter.

**Objects and Shadows:**

- a. Place various objects in the path of the torch's light.
- b. Observe how shadows are formed and note differences with transparent, translucent, and opaque objects.

**Colored Objects:**

Shine the torch on colored objects and observe how the color of the object affects the color of the light.

**Recording Observations:**

- a. Provide participants with plain paper and pencils.
- b. Ask them to record their observations, noting the behavior of light in different situations.

**Discussion:**

- a. Gather participants to discuss their observations and findings.
- b. Discuss the concepts of reflection, refraction, dispersion, and the interaction of light with different materials.

**Extension (Optional):**

- a. Experiment with additional materials and surfaces.
- b. Explore the effects of combining colored lights.

**Conclusion:**

- a. Summarize the key concepts learned during the practical activity.
- b. Discuss real-world applications of light properties.



## Points to Remember

- **GEOMETRICAL OPTICS: or ray optics**, is a model of optics that describes light propagation in terms of ray.
- **Light** is a form of electromagnetic radiation that enables the human eye to see or make things visible.
- **Propagation** of light refers to the manner in which an electromagnetic wave transfers its energy from one point to another.
- **A ray** is an idealized narrow beam or a column of light.
- **A beam** is a narrow projection of a set of particles or waves.
- Principle of reversibility of light

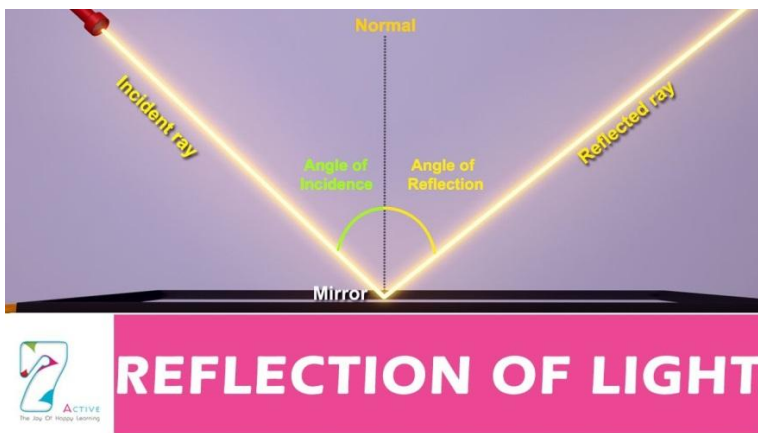
If a ray of light travel from medium 1 to medium 2 along a certain path, it retraces the path, when it passes from medium 2 to medium 1. Thus, the path of light is reversible.



## 5.2 : Application of light reflection on surfaces

### 5.2.1. Laws of reflection

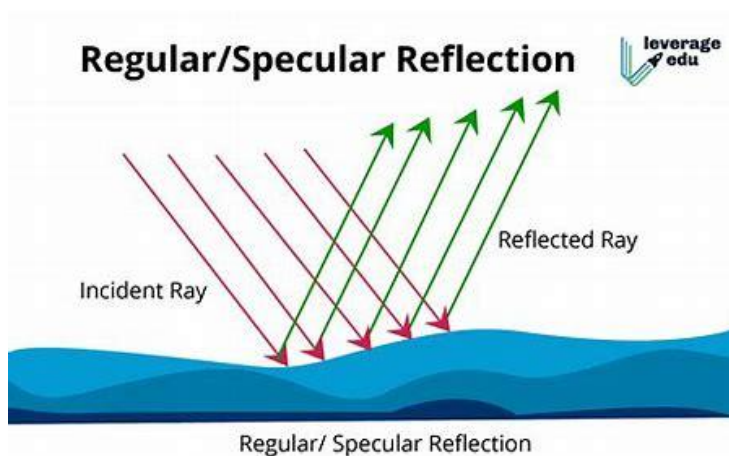
To begin with, the reflection of light occurs **whenever a ray of light falls on a smooth polished surface and bounces back.**



Different types of reflection of light are briefly discussed below:

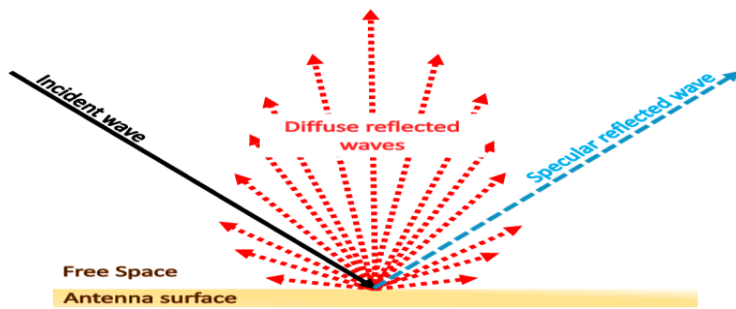
- Regular reflection is also known as specular reflection

Reflection such that the angle of reflection of the light is equal to the angle of incidence and on the opposite side of the normal to the point of incidence



- **Diffused reflection**

Diffuse reflection is the **scattering of light that occurs when it reflects off a surface.**

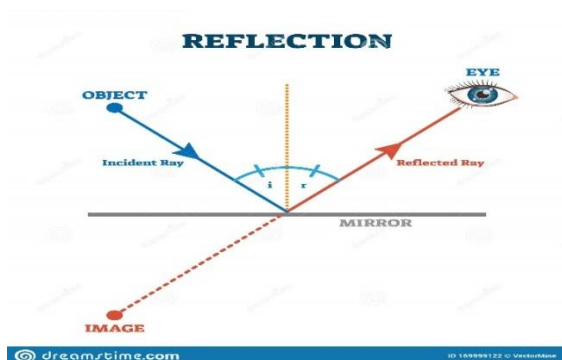


- **Multiple reflection**



Multiple reflection of light is the reflection of light back and forth through reflecting surfaces several times.

**A. Ray diagram for reflection**

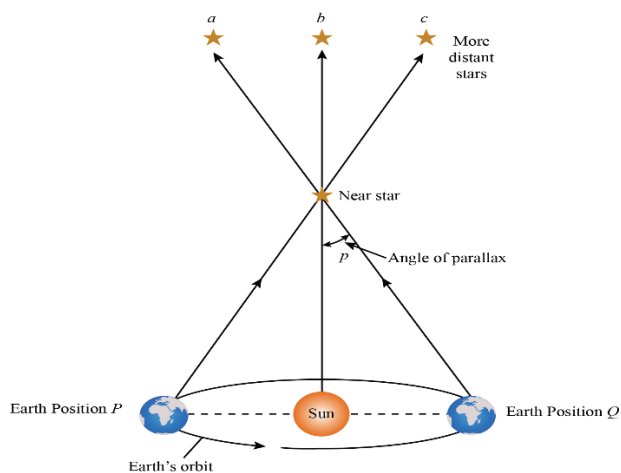


**B. Laws of reflection**

1. The law of reflection states that: angle of incidence  $i =$  angle of reflection  $r$ .
2. The second law of reflection states “the incident ray, the normal to the mirror at the point of incidence and the reflected ray, all lie in the same plane”.

### C. Parallax method

Parallax is a method based on measuring two angles and sides of a triangle formed by the star, earth on one side and other side six month later.



#### 5.2.2. Formation of image in mirrors

**Mirrors** are smooth reflecting surfaces, usually made of polished metal or glass that has been coated with a metallic substance.

A real image and a virtual image are **different forms of image**. The main difference between real and virtual images lies in the way in which they are produced. A real image is formed when rays converge, whereas a virtual image occurs where rays only appear to diverge.

#### What is the difference between real image and virtual image?

Hence, they can be captured on the screen. Conversely, there is an imaginary intersection of the rays of light in the case of virtual image, so it cannot be cast on the screen. In general, real images are inverted, whereas virtual images are erect. The converging lens is used to form a real image.

#### Interpret images

#### Terms and Definitions

● **Centre of curvature (C)** is the point in the centre of the sphere from which the mirror was sliced. For concave mirrors, the centre C, of the sphere is in front of the reflecting surface. For a convex mirror, C is behind the reflecting surface.

- **Vertex** is the point on the mirror surface where the principal axis meets the mirror. The vertex is also known as the pole. The vertex is the geometric center of the mirror.
- **Focal point (F)** is the point midway between the vertex and centre of curvature. It is also called the “principal focus”.
- **Radius of curvature(R)** is the distance between the centre of the curvature and the vertex. It is the radius of the sphere from which the mirror was cut.
- **Focal length (f)** is the distance from the mirror to the focal point.
- **Aperture** is the surface of the mirror. Since the focal point (F) is the mid-point of the line segment joining the vertex and the centre of curvature, the focal length (f) would be half the radius of curvature. i.e.  $f = R/2$  or  $R = 2f$

### Ray Diagrams

To construct the image, two of the following three rays are drawn from the top of the object:

1. A ray parallel to the principal axis which after reflection actually passes through the focal point or appears to diverge from the focal point.

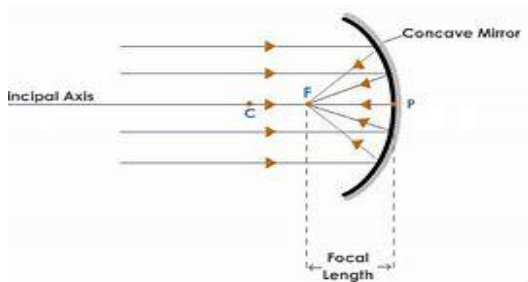


Figure 29: Rays reflecting through the main focus

2. A ray through the centre of the curvature which strikes the mirror normally and is reflected along the same path.

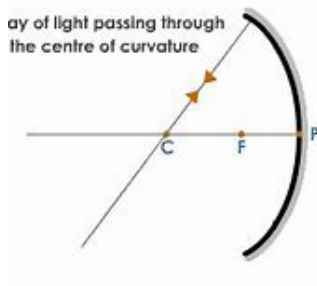


Figure 30: Rays passing through the center of curvature

3. A ray through the principal axis at the focal point which is reflected parallel to the principal axis, i.e. a ray taking the reverse path of (1).

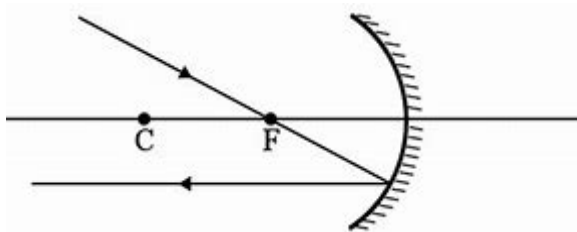


Figure 31: Rays passing through the main focus

### Image characteristics of Concave Mirrors:

#### Case 1: The object (O) is located beyond the centre of curvature

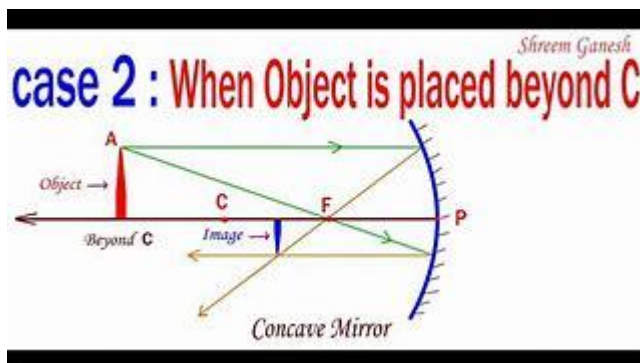


Figure 32: Image of an object placed beyond C

The image (I) is located between C and F, it is real, inverted and diminished.

**Case 2: The object (O) is located at the centre of curvature**

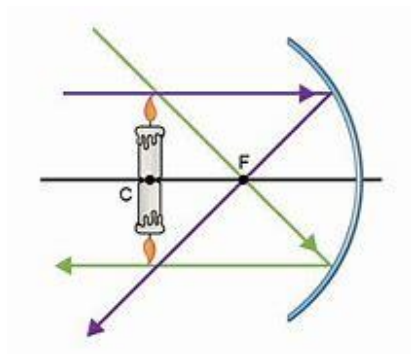


Figure 33: Object placed at C

The image (I) is located at C, it is real, inverted and the same size.

**Case 3: The object (O) is located between C and F.**

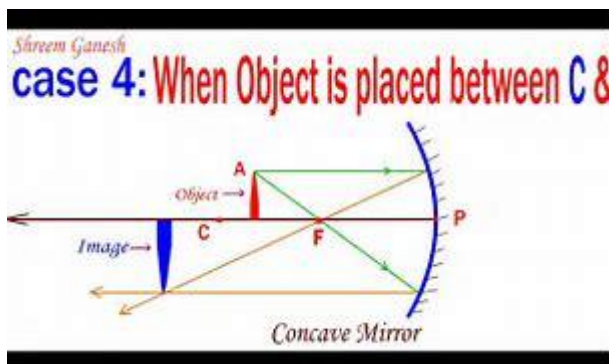


Figure 34: Image of an object placed between C and F

The image (I) is located beyond C, it is real, inverted and magnified.

**Case 4: The object (O) is located between the focal point and vertex.**

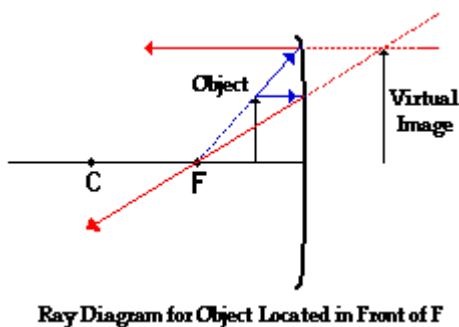


Figure 35: Image given by a convex mirror

The image (I) is located behind the mirror, it is virtual, upright and magnified.

**Case 5: The object (O) is located at the focal point.**

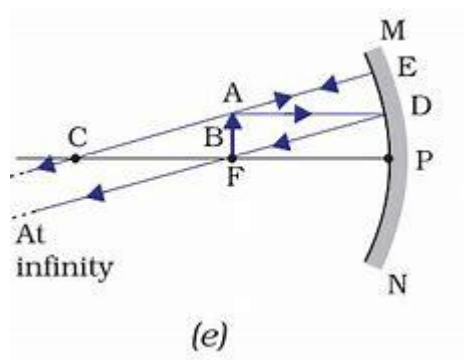


Figure 36: Image of an object placed at F is found at infinity

The image is located at infinity because the reflected rays are parallel.

**Image characteristics for Convex Mirrors:**

Consider different positions of the object (O):

The image (I) for the convex mirror is always located between F and the vertex, it is virtual, upright and diminished.

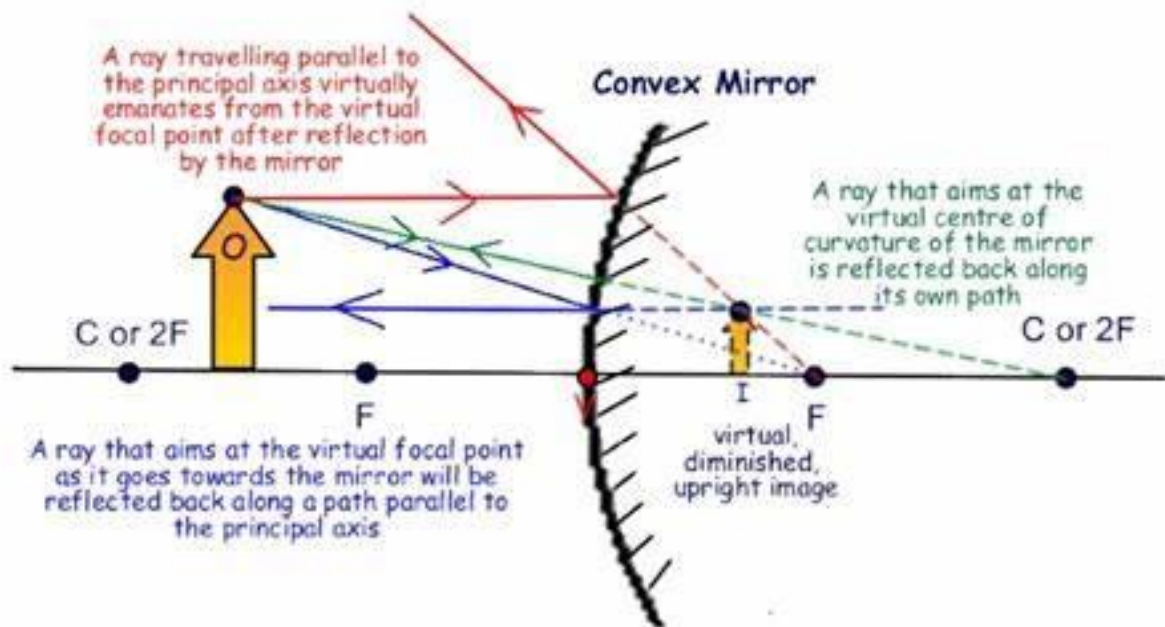


Figure 37: Image characteristic for a convex mirror

## Identify the types of mirrors

- Curved mirrors

A **curved mirror** is a mirror with a curved reflecting surface.

- Plane mirrors

A **plane mirror** is a mirror with a flat (planar) reflective surface.

- spherical mirrors

- Concave mirror (converging mirror)

**Concave mirror** is called a converging mirror because **parallel rays of light fall on the mirror they converge at a point called focus**

- Convex mirror (diverging mirrors)

**Convex mirror** is called a diverging mirror because parallel rays of light fall on it they diverge after reflection.

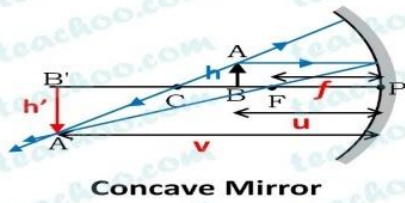


## Magnification Equation

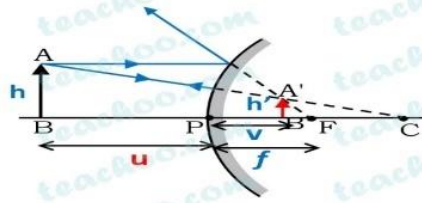
- The magnification ( $m$ ) tells you **the size**, or **height** of the image relative to the object, using object and image distances.
  - Therefore, in order to use this equation the **distance** of the object and image must be known.

$$m = \frac{h_i}{h_o} = \frac{-d_i}{d_o}$$

## Mirror Formula and Magnification



Concave Mirror



Convex Mirror

**Mirror Formula:**

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

**Magnification**

$$m = \frac{\text{Height of image}}{\text{Height of Object}}$$

$$m = \frac{-v}{u}$$

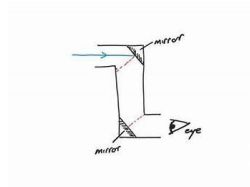
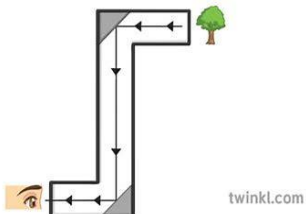
### 5.2.3. Applications of light reflection



A **periscope** is defined as an instrument used for observing over, around or through an obstacle or object which is prevented by direct line of sight

#### Reflecting periscope

In periscope, we use the **totally reflecting prisms which turn the ray through 90°**. A totally reflecting prism is that which has one of its angle equal to 9° and each of the remaining two angles equal to 45°.



#### Car side mirrors

The **side view mirror** is responsible for giving you a clear view of what's happening around the vehicle, allowing you to make good decisions when changing lanes or pulling out into traffic.



### Theoretical learning Activity

**In group four, trainees discuss the following:**

1. A ray of light is incident towards a plane mirror at an angle of  $30^{\circ}$  with the mirror surface. What will be the angle of reflection?
2. A concave spherical mirror has a focal length of 10.0cm. Locate the image of a pencil that is placed upright 30.0cm from the mirror.
  - a) Find the magnification of the image.
  - b) Draw a ray diagram to confirm your answer.
3. What is diffuse reflection?
4. What are the laws of reflection?
5. What is parallax?
6. Define what is a mirror
7. Identify the types of mirrors
8. Define what is periscope



### Practical learning Activity

Reflection in spherical mirror

### Materials:

- Concave mirror (s)
- Convex mirror (s)
- An object like a candle

### Procedures:

- Observe the images of your objects (candle) using given mirrors.
- Move the candle near by the mirror or far from the mirror and observe the changes on its images.

### Questions:

1. What happens to the image of the candle as it moves near to the curved mirror?
2. What happens to the image of the candle as it moves far from the curved mirror?
3. Discuss other cases where you observe such situations.



### Points to Remember

The reflection of light occurs whenever a ray of light falls on a smooth polished surface and bounces back.

Diffuse reflection is the scattering of light that occurs when it reflects off a surface.

Multiple reflection of light is the reflection of light back and forth through reflecting surfaces several times.

#### Laws of reflection

1. The law of reflection states that: angle of incidence  $i$  = angle of reflection  $r$ .
2. The second law of reflection states “the incident ray, the normal to the mirror at the point of incidence and the reflected ray, all lie in the same plane”.

**Mirrors** are smooth reflecting surfaces, usually made of polished metal or glass that has been coated with a metallic substance

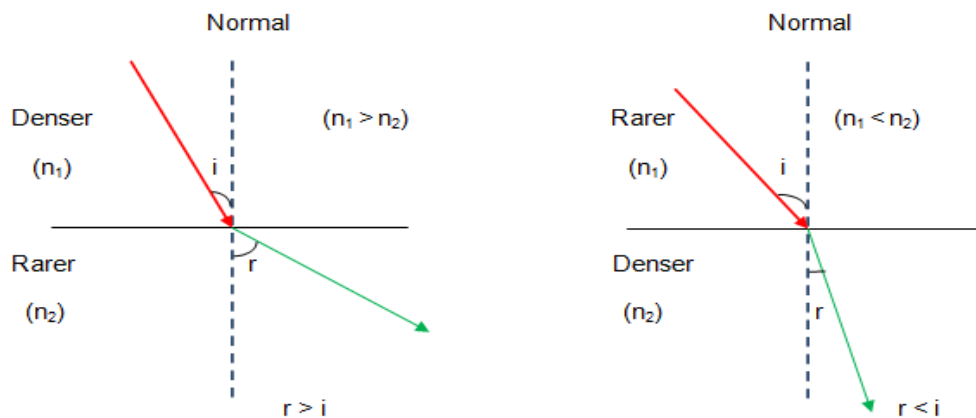


## 5.3 : Application of light refraction in different media

### 5.3.1. Key terms

In the physical sciences, an **interface** is a boundary between two regions of space occupied by different matter, or by matter in different physical states. The interface between matter and air, or matter and vacuum, is called a surface and is studied in surface science.

**Refraction**, in **Physics**, the change in direction of a wave passing from one medium to another caused by its change in speed. For example, waves travel faster in deep water than in shallow.



### 5.3.2. Laws of refraction

1. The incident ray, the refracted ray and the normal, at the point of incidence, all lie in the same plane.
2. The ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant for a given pair of media (Snell's law) i.e.  $\frac{\sin\theta_1}{\sin\theta_2} = \text{constant}$

The expression  $\sin \theta_1 / \sin \theta_2 = \text{constant}$  is the mathematical expression of Snell's law.

## Refractive index

The refractive index ( $\eta$ ) is the measure of bending of light i.e is the ratio of sine of angle of incident to the sine of angle of refraction (hence Snell's law).

Snell's law can also be stated as  $\sin i \sin r = \text{a constant}$ .

This constant is known as *refractive index*,  $\eta$  or *index of refraction*. Therefore,

Refractive index ( $\eta$ ) =  $\frac{\sin\theta_1}{\sin\theta_2}$  It has no units

**Example:** light travels from air to glass the ratio  $\frac{\sin\theta_1}{\sin\theta_2}$  is found to be 1.50. This is the refractive index of glass with respect to air. We can rewrite this using the following symbols.

$$\eta = 1.50 \text{ i.e., } \eta_g = 1.50$$

The absolute refractive index of a medium is the value of  $\eta$  when the first medium is a vacuum.

The absolute refractive index of a medium  $\eta = \frac{\sin\theta_1}{\sin\theta_2}$

Where the angle of incidence is in vacuum and the angle of refraction is in the medium

### Example

A ray of light passing from air to glass is incident at an angle of  $30^\circ$ . Calculate the angle of refraction in the glass, if the refractive index of glass is 1.50.

### Solution

$$\text{Refractive index of glass } \eta_g = \sin \theta_1 / \sin \theta_2$$

$$\therefore \sin \theta_2 = \sin \theta_1 / \eta_g = \sin 30^\circ / 1.50 = 0.50 / 1.50 = 0.33$$

$$\therefore \theta_2 = 19.5^\circ$$

The angle of refraction in glass is  $19.5^\circ$

### 5.3.3. Real and apparent depth

**The real depth** is the actual depth of the bottom of the tank and the apparent depth is the virtual depth that is observed as a result of the refraction of light.

**The apparent depth** depends upon the refractive index of the medium. The refractive index is a measure of the bending of the ray as it passes from one medium to another.

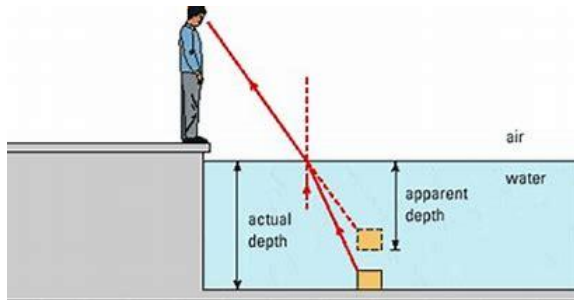


Figure 38: Real and apparent depth

### 5.3.4. Critical angle and total internal reflection

For light to refract from a more optical dense to a less optical dense medium, the light ray must be incident at an angle less than a certain angle. The following activity will help us to determine this angle.

As the angle of incidence increases, the ray is refracted further away from the normal.

**The critical angle** is the angle of incidence in a denser medium for which the angle of refraction is  $90^\circ$  in the rarer medium.

**Total internal reflection** is a phenomenon that occurs when a light ray traveling through a medium with a higher refractive index encounters a boundary with a medium having a lower refractive index, and the angle of incidence is greater than the critical angle. Under these conditions, the light is completely reflected back into the higher refractive index medium, with no transmission into the lower refractive index medium.

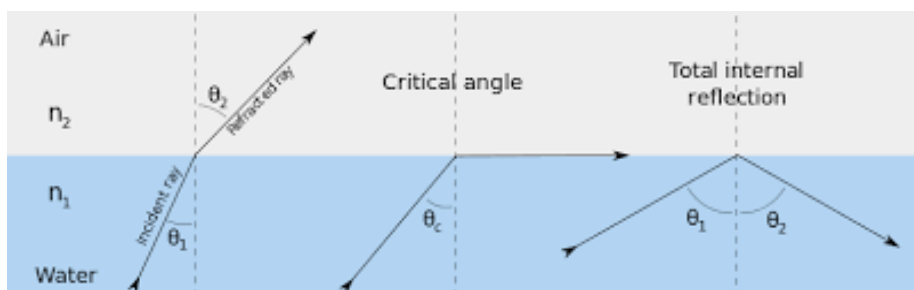


Figure 39: critical angle and total internal reflection

### 5.3.5. Refraction of light

#### Refraction of light in prism

The phenomenon of splitting of white light into **its seven constituent colours** when it passes through a glass prism is called dispersion of white light. The various colours seen are Violet, Indigo, Blue, Green, Yellow, Orange and Red.

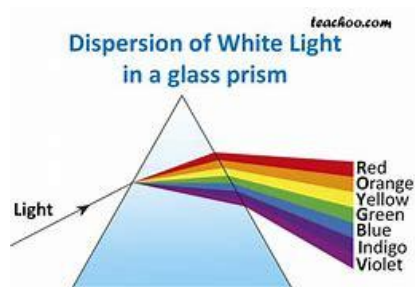


Figure 40: Dispersion of light.

A *monochromatic light* is one that has a *single colour* and a *single frequency* or *single wavelength*. White light is not monochromatic because it is made up of seven different colours. Non-monochromatic light is also called *composite light*.

For the same angle of incidence, each colour inside the glass prism has its own angle of refraction and angle of deviation. Since refractive index is given by  $n = \frac{\sin i}{\sin r}$ , it follows that each colour has its own refractive index for glass. But refractive index is also given by  $n = \frac{\text{speed of light in air}}{\text{speed of light in glass}}$ .

#### Refraction of light in thin lenses

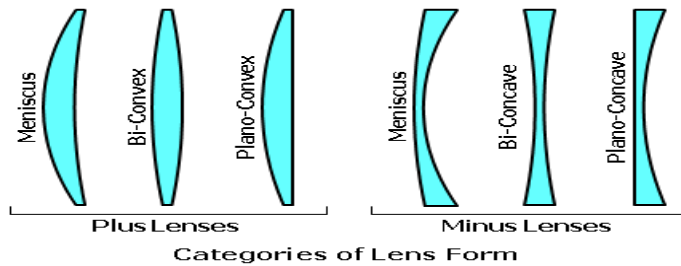
A **lens** is a transparent medium bounded by two spherical surface or a planned curved surface.

#### Types of lenses

There are two main groups of lenses. A type that is thick in the middle and thin at the edges, causing rays of light to converge. This is called **converging or convex lenses**. The other type is thin in middle and thick at the edges causing the rays of light to diverge. This lens is called **diverging or concave lens**.

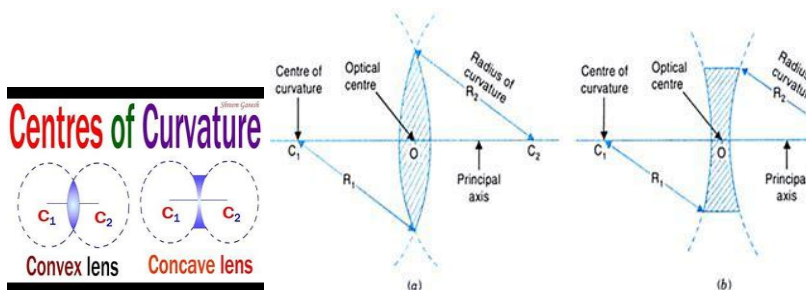
A **bi-convex** or **double convex** lens has both its surfaces 'curving out'.

A **bi-concave** or **double concave** lens has both its surfaces 'curving in'. Other concave lenses are plano-concave and convexo-concave or diverging meniscus



### Terms used in thin lenses

#### (a) The centre of curvature (C)



The centre of curvature of the surface of a lens is the centre of the sphere of which the surface forms a part. For each spherical lens there are two centres of curvature ( $C_1$ ,  $C_2$ ) due to the two curved surface.

#### (b) The radius of curvature (r)

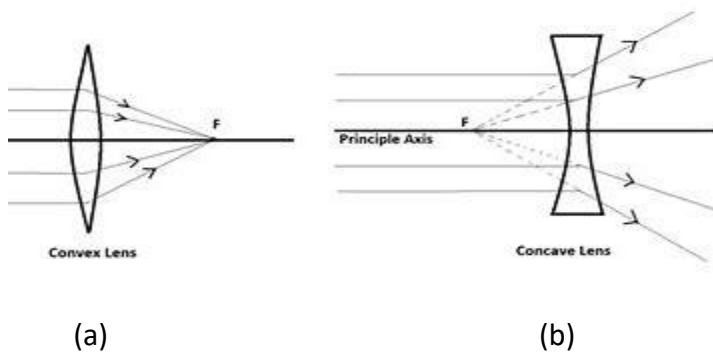
The radius of curvature of the surface of a lens is the radius of the sphere of which the surface forms a part. Each surface has its own radius of curvature ( $r_1$  or  $r_2$ ).

#### (c) Principal axis

The principal axis of a lens is a line passing through the two centres of curvature ( $C_1$  and  $C_2$ ).

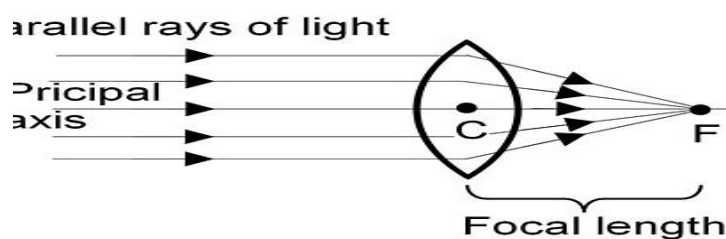
#### (d) The principal focus

A prism always deviates the light passing through it towards its base. A convex lens may be regarded as being made up of large portions of triangular prisms as shown below. The emergent beam, therefore, becomes convergent in a convex lens (Fig a). The reverse is the effect in a concave lens (Fig. b).



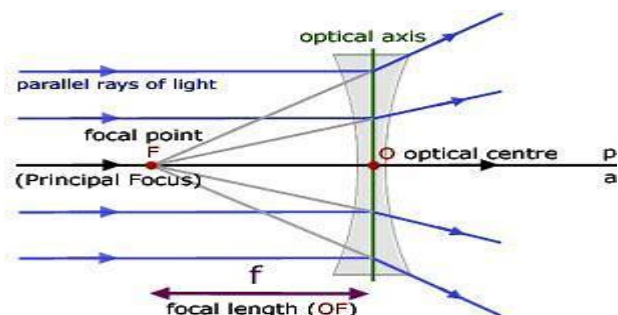
**i. Principal focus of a convex lens**

Consider a set of incident rays parallel and close to the principal axis of a convex lens. These rays, after refraction through the lens, pass through point **F** on the principal axis. Since all the rays converge at this point, it is called **principal focus**. Since this point can be projected on a screen, it is said to be a real principal focus.



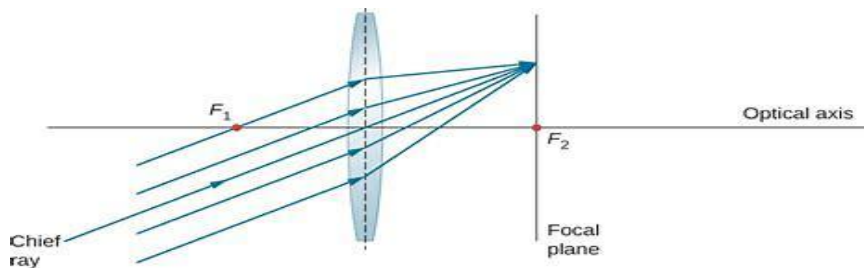
**ii. Principal focus of a concave lens**

For a set of incident rays parallel and close to the principal axis of a concave lens, the refracted rays appear to diverge from a fixed point on the principal axis. This point is called **the principal focus F**, of a concave lens. This principal focus is virtual since it cannot be projected on a screen.



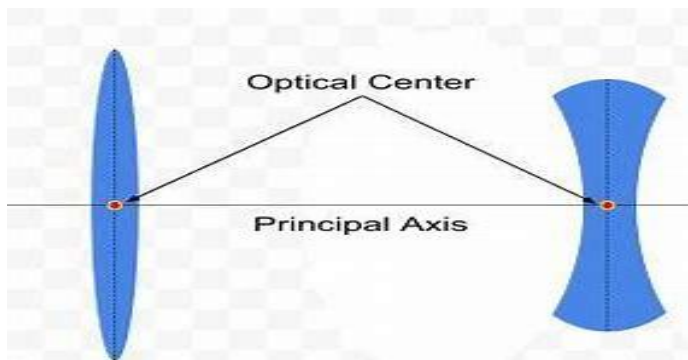
### (e) The focal plane

When a set of parallel rays are incident on a convex lens at an angle to the principal axis, as shown in Fig. below, the refracted rays converge to a point, on a line passing through  $F$  and perpendicular to the principal axis. The plane passing through  $F$  is **the focal plane**.



### (f) The optical centre (P)

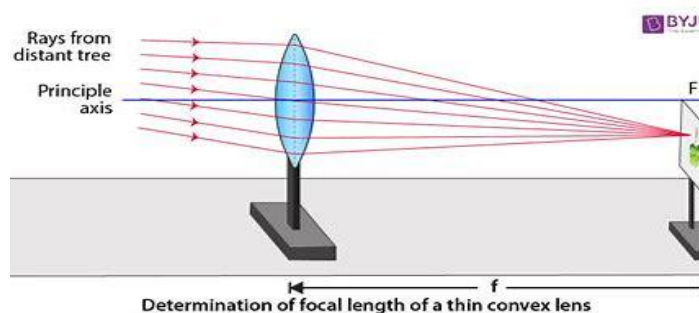
The optical centre of a lens is a point which lies exactly in the middle of the lens as shown in Fig. below. Light rays going through this point go straight through without any deviation or displacement.



### (g) The focal length of a lens (f)

This is the distance from the optical centre to the principal focus of the lens.

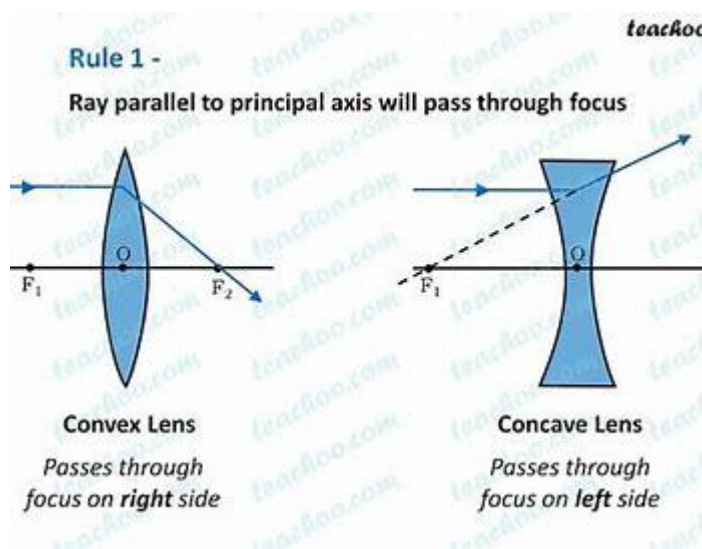
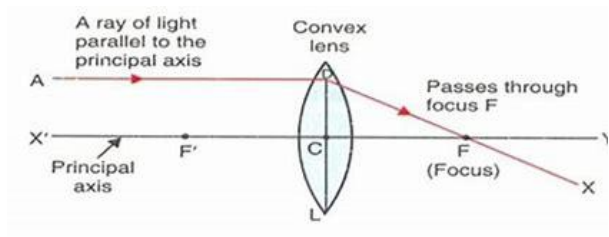
Biconvex and biconcave lenses have a focal length on each side of the lens.



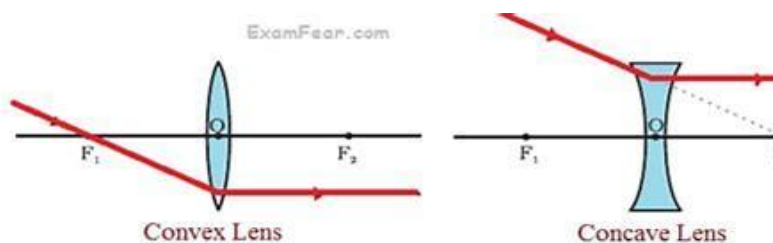
## Image formation by converging lenses

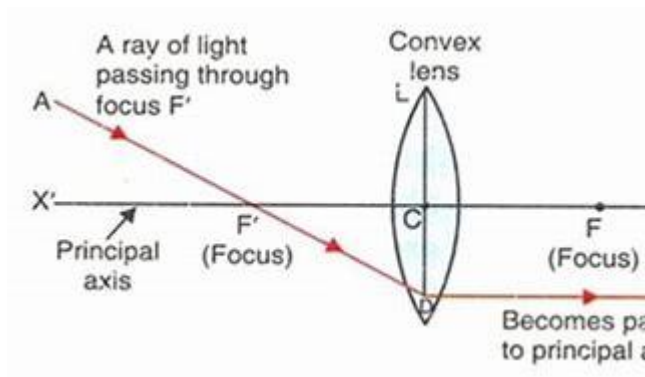
The following are the important incident rays and their corresponding refracted rays used in the construction of ray diagrams.

**Ray 1:** A ray of light parallel and close to the principal axis, passes through the principal focus F

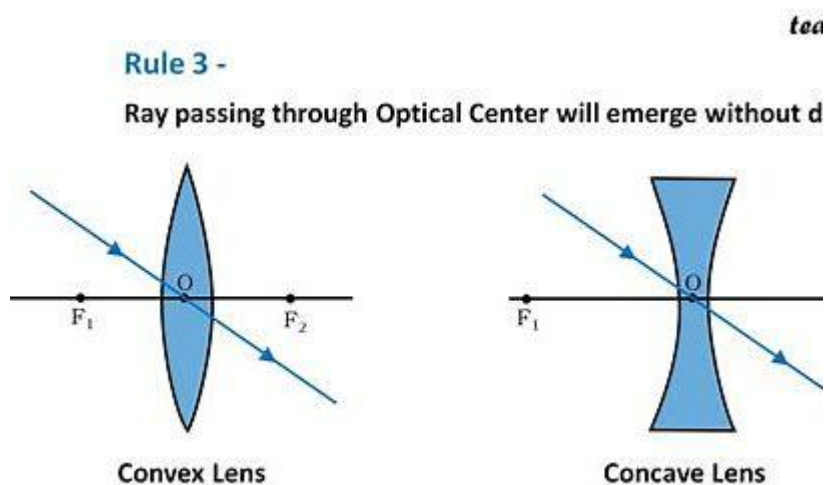


**Ray 2:** A ray of light through the principal focus F emerges parallel to the principal axis after refraction





**Ray 3:** A ray through the optical centre, P is undeviated after refraction through the lens



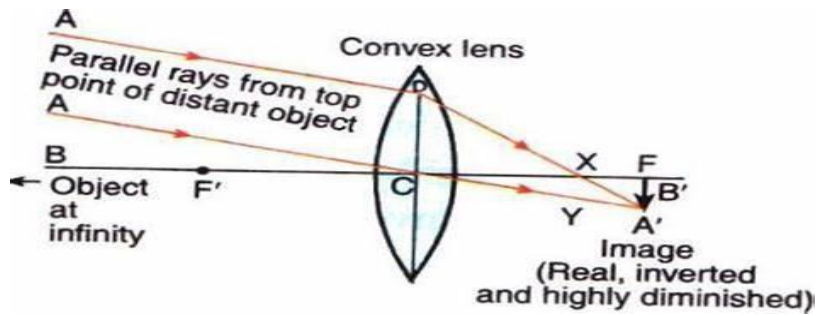
### Locating images by simple ray diagrams and describing their character

To locate the image of an object, we need a minimum of two incident rays from the object. From the three standard rays discussed above, any two incident rays and their corresponding refracted rays can be drawn to locate the image. If the refracted rays converge, a **real image** is obtained. If the refracted rays diverge, then a **virtual image** is obtained.

#### A. Convex lens

##### (a) Object far away from the lens (at infinity)

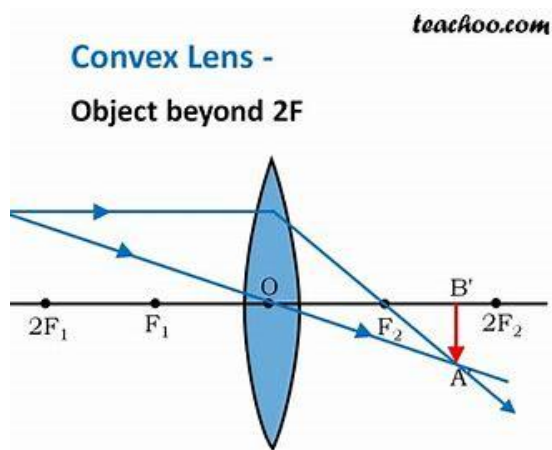
Since the object is at infinity, all the rays from the object, incident on the lens are almost parallel. The refracted rays converge at a point on the focal plane, as shown in Fig. below.



**Image characteristics**

A diminished, real, inverted image is formed at F.

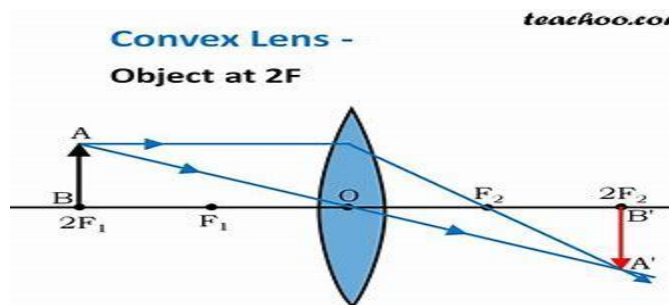
**(b) Object OB just beyond C (2F)**



**Image characteristics**

A diminished, real, inverted image is formed between F and 2F.

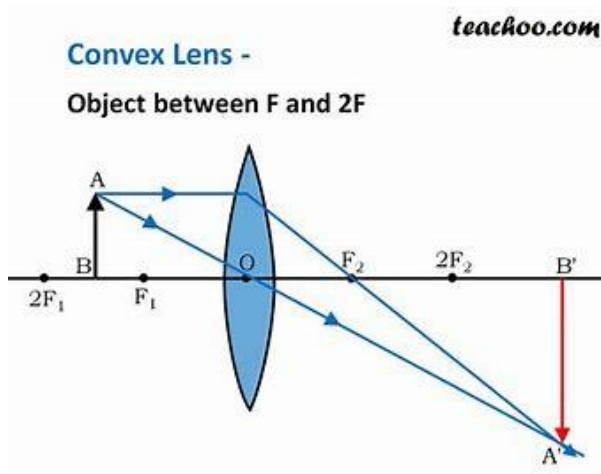
**(c) Object OB at 2F**



**Image characteristics**

A real, inverted image of the same size as the object is formed at 2F

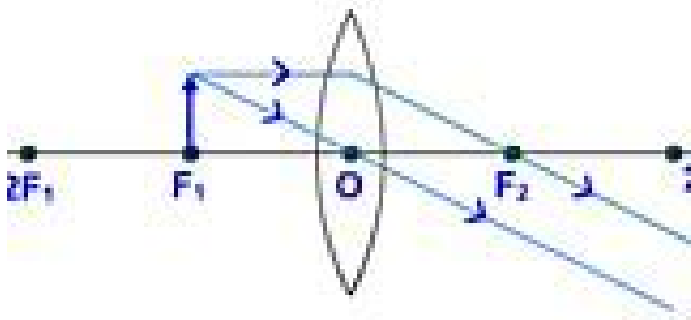
**(d) Object OB between 2F and F**



**Image characteristics**

A real, inverted and magnified image is formed beyond 2F

**(e) Object OB at F**



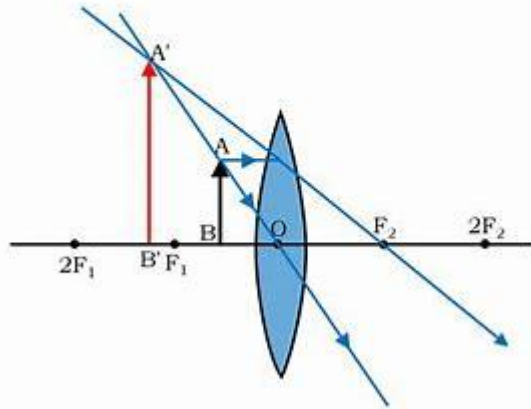
**Image characteristics**

A real, inverted, magnified image is formed far away from the lens i.e. at infinity. (can not be described)

**(f) Object OB between F and P**

**Convex Lens -**

**Object between Optical Center and Focus (O and F)**

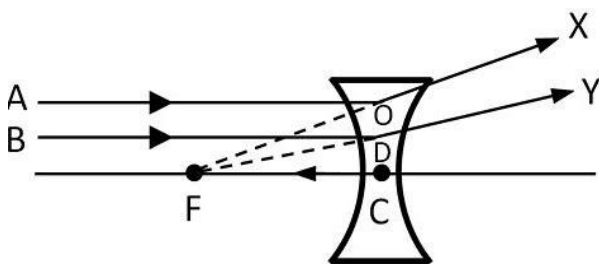
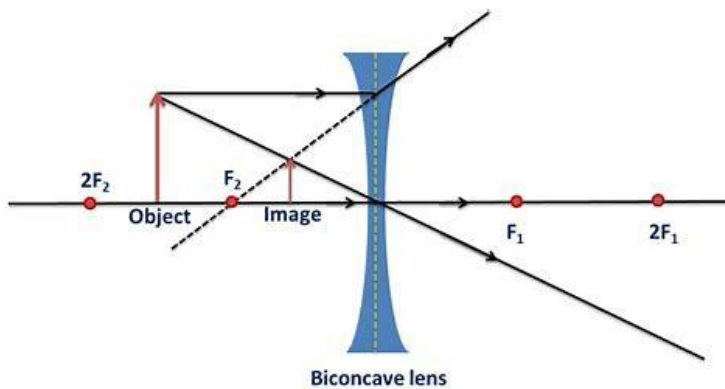


**Image characteristics**

A magnified, upright and virtual image is formed on the same side as object.

**Concave lens**

When the object is at infinity, an upright, diminished and virtual image is formed at principal focus F. For all other positions of the object OB, an upright, diminished, virtual image is always formed between F and P



**The lens formula**

The lens formula is a formula relating the focal length, image and object distance. Consider a convex lens of focal length,  $f$ ,

## The lens formula

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$f$  = focal length (m)  
 $u$  = object distance (m)  
 $v$  = image distance (m)

### Sign Convention

1. All the distances are measured from the optical centre.
2. The distances of the real objects and the real images measured from the optical centre are taken as **positive**, while those of virtual objects and virtual images are taken as **negative**. From this convention, **the focal length of a convex lens is positive** and that of a **concave lens is negative**.

### Magnification formula of the lens

The term magnification refers to how many times an image is bigger than the object. Linear magnification ( $m$ ) is defined as the ratio of the height of the image to the height of the object.

$$\text{Magnification (m)} = \frac{\text{image distance (v)}}{\text{object distance (u)}}$$

$$\text{Or } m = \frac{v}{u}$$

Therefore  $m = \frac{h_i}{h_o} = \frac{v}{u}$ , therefore, the ratio of image to object sizes  $h_i / h_o$  is also equal to the ratio of image to object distances  $v/u$  measured from the optical

### Power of a lens

The ability to collect rays of light and focus them at a point in the case of a converging, or to diverge them so that they appear to come from a point in the case of diverging lens is called the power of a lens. It is calculated from its focal length using the formula

$$\text{Power} = \frac{1}{f}$$

The unit for power is the dioptre represented by the symbol D. The  $f$  must be in S.I units of length.

### Example

A lens has a focal length of 25 cm. Find the power of the lens.  $f = 25 \text{ cm} = 0.25 \text{ m}$ .

The focal length of convex lens = +ve (It forms real image)

$$\therefore \text{Power} = 1/0.25 = +4 \text{ m}^{-1}$$

NB: For a concave lens  $f = -\text{ve}$  (because a concave lens forms a virtual image)

$$\therefore \text{Power} = 1/-0.25 = -4 \text{ m}^{-1}$$



### Theoretical learning Activity

**In pairs, trainees do the following task:**

1. An object is placed 24 cm from the centre of a convex lens of focal length 20 cm. Calculate the distance of the image from the lens.
2. An object is placed 12 cm from the centre of a concave lens of focal length 20 cm. Calculate the distance of the image from the lens.
3. An object of height 2 cm is placed 20 cm in front of a convex lens. A real image is formed 80 cm from the lens. Calculate the height of the image.
4. An object placed 30 cm from a convex lens produces an image of magnification 1. What is the focal length of the lens?
5. An object of height 1.2 cm is placed 12 cm from a convex lens and real image is formed at 36 cm from the lens. Calculate (a) the focal length of the lens (b) magnification produced by the lens (c) the size of the image
6. An object of height 2 cm is placed 8 cm from a convex lens and a virtual image is formed on the same side as the object at 24 cm from the lens. Calculate (a) the focal length of the lens (b) the height of the image formed.



## Practical learning Activity

### Materials

- Plastic ruler
- Water in a transparent container

### Steps

1. Dip a plastic ruler into a transparent container of clean water
2. View the ruler from the top and the side of the container (Fig. 5.). What do you observe on the shape of the ruler? Explain.

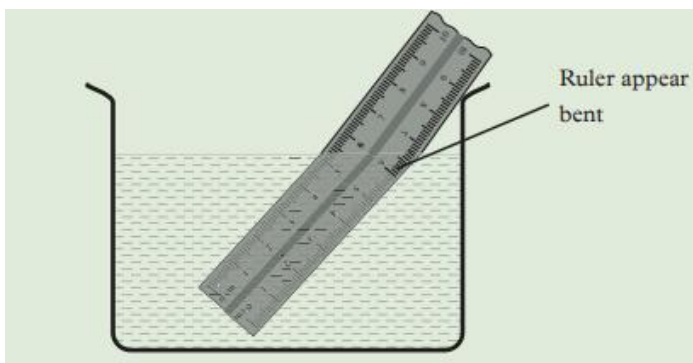


Figure 41: Appearance of a ruler in water.



### Points to Remember.

- The medium with greater refractive index is called the denser medium and the one with smaller refractive index is called the rarer medium.
  - The following conditions must be satisfied for total internal reflection to occur:
    1. Light must travel from a denser medium to a rarer medium.
    2. The angle of incidence in the denser medium must be greater than the critical angle.
- The phenomenon of splitting of white light into its seven constituent colours when it passes through a glass prism is called dispersion of white light.
- A *monochromatic light* is one that has a *single colour* and a *single frequency* or *single wavelength*.
  - A **lens** is a transparent medium bounded by two spherical surface or a plane curved surface.
  - There are two main groups of lenses. A type that is thick in the middle and thin at the edges, causing rays of light to converge. This is called **converging or convex lenses**. The other type is thin in middle and thick at the edges causing the rays of light to diverge. This lens is called **diverging or concave lens**.
  - The term magnification refers to how many times an image is bigger than the object.



## Learning outcome 5. Formative Assessment

### Written assessment

1. Distinguish between converging and diverging lenses.

### Solution

#### **Converging Lens:**

- Thicker in the center and thinner at the edges.
- Converges (brings together) parallel incident light rays.
- Forms real or virtual images depending on the object distance.

#### **Diverging Lens:**

- Thinner in the center and thicker at the edges.
  - Diverges (spreads out) parallel incident light rays.
  - Always forms virtual images.
2. Define the following: (a) Principal axis (b) Optical centre

### Solution

#### **Principal Axis:**

- Imaginary line passing through the optical center and the focal point of a lens or mirror.
- It is the central axis around which optical elements are symmetrically arranged.

#### **Optical Centre:**

- Point at the center of a lens or mirror.
  - Light passing through this point does not experience deviation.
  - It is a point of reference for optical measurements and calculations.
3. a . Differentiate between the principal focus of the concave and convex lens.  
b. How many principal foci does a biconcave lens have?

## Solution

### **a. Principal Focus of Concave Lens:**

- For a concave lens, the principal focus is virtual.
- Light rays diverge after passing through the lens.
- It is on the same side as the incident light.

### **Principal Focus of Convex Lens:**

- For a convex lens, the principal focus is real.
- Light rays converge after passing through the lens.
- It is on the opposite side of the incident light.

### **b. Biconcave Lens Principal Foci:**

- A biconcave lens has two virtual principal foci.
- Light rays diverge as if coming from two points on the same side as the incident light.

4. Define the term:

a) Refraction of light

(b) Angle of incidence

## Solution

### **Refraction of Light:**

- Bending of light as it passes from one medium to another with a different optical density.
- Results in a change in the direction of light.

### **Angle of Incidence:**

- The angle formed between the incident light ray and the normal (perpendicular) to the surface at the point of incidence.
- It is a measure of how steeply or shallowly light strikes a surface.

5. Explain why light bends when it travels from one medium to another.

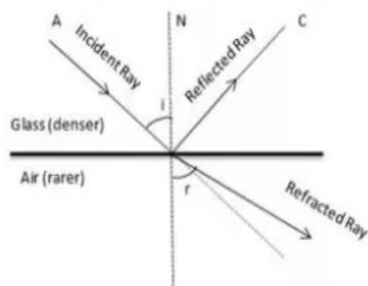
### Solution

Light bends when it travels from one medium to another due to a change in the speed of light in different media.

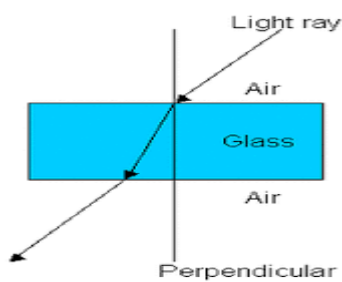
6. Draw diagrams to illustrate refraction for a ray of light on:
  - (a) Glass – air boundary
  - (b) air-glass-air boundaries
  - (c) Water – glass boundary

### Solution

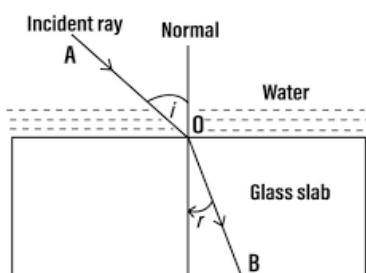
- (a) Glass – air boundary



- (b) air-glass-air boundaries



- (c) Water – glass boundary



7. Define the term refractive index.

**Solution**

- A measure of how much light is bent or refracted when it enters a medium from a vacuum.
  - It is the ratio of the speed of light in a vacuum to the speed of light in the medium.
8. State the laws of refraction.

**Solution**

- The incident ray, the refracted ray, and the normal to the boundary at the point of incidence all lie in the same plane.
  - The ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant for a given pair of media.
9. Describe an experiment to determine the refractive index of a rectangular glass block.

**Solution**

To determine the refractive index of a rectangular glass block, you can perform the following experiment:

**Materials:**

- ✓ Rectangular glass block
- ✓ Protractor
- ✓ Ray box or laser
- ✓ White paper or screen
- ✓ Drawing compass
- ✓ Ruler
- ✓ Pencil

### Procedure:

- ✓ Place the glass block on a flat surface.
- ✓ Draw a straight line AB on the paper.
- ✓ Position the glass block so that one of its long sides is along the line AB.
- ✓ Shine a ray of light through the glass block, allowing it to strike the paper.
- ✓ Trace the incident ray as it enters the glass block. Mark the point of entry as point C.
- ✓ Trace the refracted ray as it exits the glass block. Mark the point of exit as point D.
- ✓ Measure the angle of incidence ( $i$ ) between the incident ray AC and the normal to the glass-air boundary.
- ✓ Measure the angle of refraction ( $r$ ) between the refracted ray BD and the normal to the glass-air boundary.
- ✓ Repeat the experiment for different angles of incidence.
- ✓ Use Snell's Law ( $n_1 \sin(i) = n_2 \sin(r)$ ) to calculate the refractive index of the glass block. Here,  $n_1$  is the refractive index of air (approximately 1), and  $n_2$  is the refractive index of the glass block.

**Analysis:** Plot a graph of  $\sin(i)$  against  $\sin(r)$ . The slope of the graph is equal to  $n_2/n_1$ , and therefore, you can determine the refractive index ( $n_2$ ) of the glass block.

- Ensure accurate measurements and repeat the experiment to improve reliability.

10. A ray of light is passing from air into water along PQ. The ray strikes the bottom surface at T instead of R as shown in Fig. below. Calculate:

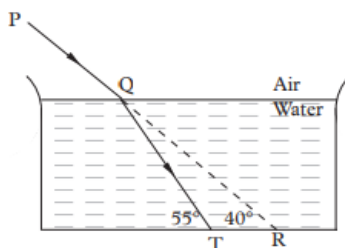


Figure 42: A ray passing air-water interface

- (a) The angle of incidence
- (b) The angle of refraction
- (c) The refractive index of water.

11. Light travels through glass of refractive index 1.60 with a speed of  $v$  m/s. calculate the value of  $v$ , if the speed of light in air is  $3.0 \times 10^8$  m/s.

**Solution**

12. The speed of light in a medium is related to the speed of light in a vacuum (such as air) and the refractive index ( $n$ ) of the medium by the formula:

$$v = c/n$$

In this case, the glass has a refractive index ( $n$ ) of 1.60, and the speed of light in air ( $c$ ) is  $3.0 \times 10^8$  m/s.

Substitute these values into the formula:

$$v = \frac{3.0 \times 10^8}{1.60}$$

Calculate the value of  $v$ :

$$v \approx 1.875 \times 10^8 \text{ m/s}$$

So, the speed of light through the glass is approximately  $1.875 \times 10^8$  m/s.

Table below shows the angles of incidence  $i$  and the angles of refraction,  $r$ , when light passes from air to glass. Complete the table and draw a graph of  $\sin r$  (y-axis) against  $\sin i$  (x-axis). From the graph determine the refractive index of glass.

$i^\circ$	$r^\circ$	$\sin i$	$\sin r$
15	10		
30	19		
45	28		
60	35		

**Solution**

1.  $n \approx 1.49$
2.  $n \approx 1.54$
3.  $n \approx 1.50$
4.  $n \approx 1.47$

## Practical assessment

### Materials

- White plain paper
- Four pins
- A ruler
- Soft board
- A glass block

### Steps

1. Fix a white plain paper on a soft board. Draw a line XY and mark its midpoint Q. At Q draw a normal NQM perpendicular to XY and a line PQ such that the angle of incidence,  $i$ , ( $\angle PQN$ ) is  $20^\circ$
2. Place a rectangular glass block such that the midpoint of the edge AB coincides with the midpoint Q of the line XY. Draw the outline of the glass block ABCD.
3. Stick two pins  $O_1$  and  $O_2$ , called the object pins on the straight-line PQ vertically into the soft board about 5.0 cm apart. View from the side CD and look for the images of the first two pins.
4. Keeping the eye along the plane of the paper, move the head to and fro slowly until in one particular position the images of the two pins lie on a straight line.
5. Fix a third pin  $S_1$  called the search pin, such that this pin and the images of the first two pins as seen through the glass block lie along the same straight line.
6. Repeat the above procedure with the fourth pin  $S_2$ , so that the images of the pins  $O_1$  and  $O_2$  and the search pins  $S_1$  and  $S_2$  lie along the same straight line.
7. Using a pencil, mark the positions of the four pins with a small circle and remove the pins and the glass block.
8. Join the points  $S_2$  and  $S_1$  to meet the line DC at R. Join points  $O_2$  and  $O_1$  to meet at Q. Join Q to R to make line QR. QR is the refracted ray in the glass for the incident ray PQ in air. Measure the angle of refraction,  $r$ , ( $\angle MQR$ ).

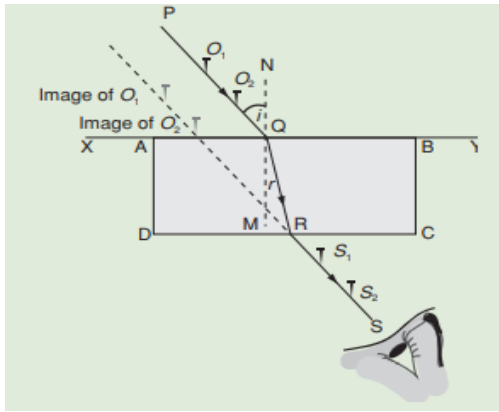
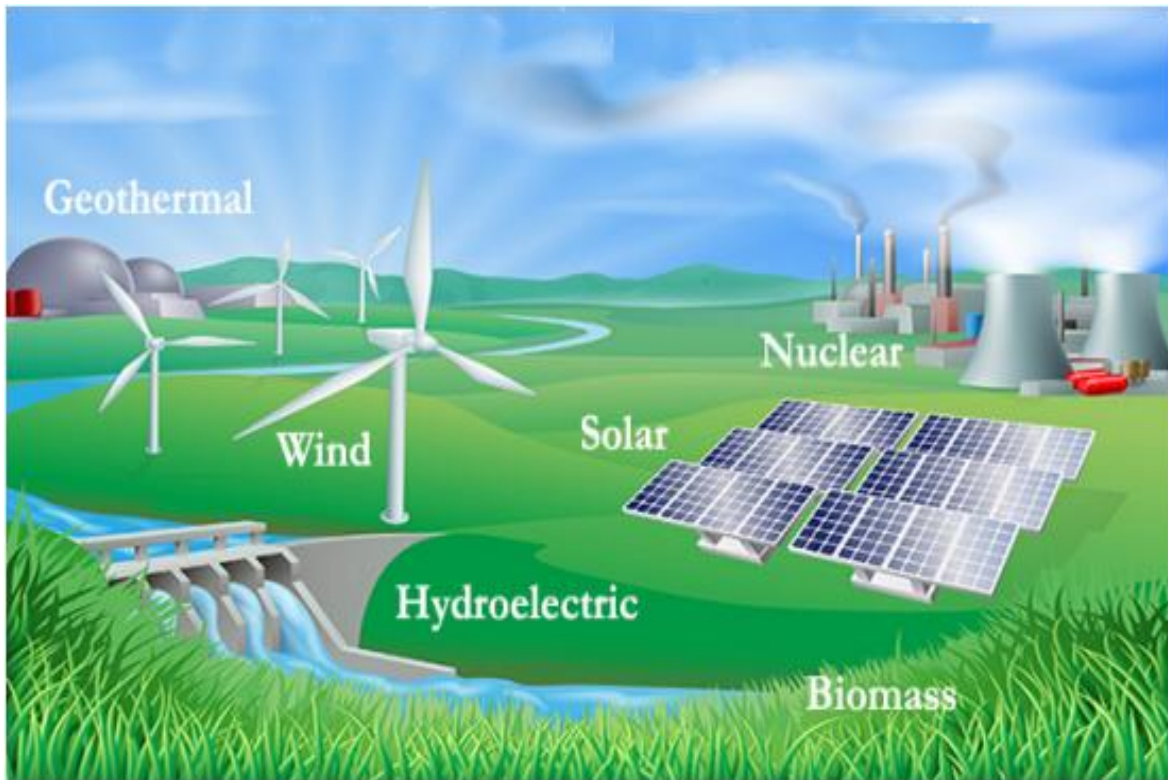


Figure 43: Verification of Snell's law.

9. Repeat the experiment for different angles of incidence and record the readings in a table similar to Table 12.1.
10. What do you observe on the column  $\frac{\sin i}{\sin r}$ ? What does it represent?

## Learning outcome 6: Characterize sources of energy in the world



## Learning outcome 6. Characterize sources of energy in the world

### Indicative contents:

- 6.1. Explanation of basic concepts of work, energy and power
- 6.2. Identification of types of energy
- 6.3. Analysing relative advantages and disadvantages of various energy sources



Duration: 6 hours



### Learning outcome 6 objectives:

By the end of the learning outcome, the trainees will be able to:

1. Identify properly types of energy according to modes of extraction and creation of energy systems in line with sources
2. Analyse properly relative advantages and disadvantages of various energy sources according to the sources
3. Calculate efficiently linear momentum and kinetic energy in collision based on the laws of conservation



### Resources

Equipment	Tools	Materials
- PPE, whiteboard and chalkboard, computer, projector, textbooks	- Scientific calculator	- Chalks, markers



### Advance preparation:

- Prepare in advance the learning room, and task sheets to be used.
- Prepare relevant pictures that reflect work energy and power.
- Prepare simulations and videos on energy creation.



## 6.1 : Explanation of basic concepts of work, energy and power

### 6.1.1. Explanation of work:

In Physics, **work** is the energy transferred to or from an object via the application of force along a displacement.

Mathematically, the concept of work done  $W$  equals the force  $f$  times the distance ( $d$ ), that is  $W = f \cdot d$  and if the force is exerted at an angle  $\theta$  to the displacement, then work done is calculated as  $W = f \cdot d \cos \theta$ .

### Unit of Work

The SI unit of work is Joule (**J**).

For example, if a force of 5N is applied to an object and moves 2 meters, the work done will be 10 newton-meter or 10 Joule. It should be noted that  $1 \text{ J} = 1 \text{ N} \cdot \text{m} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2$ .

### Example of Work

An object is horizontally dragged across the surface by a 100 N force acting parallel to the surface. Find out the amount of work done by the force in moving the object through a distance of 8 m.

### Solution:

#### Given:

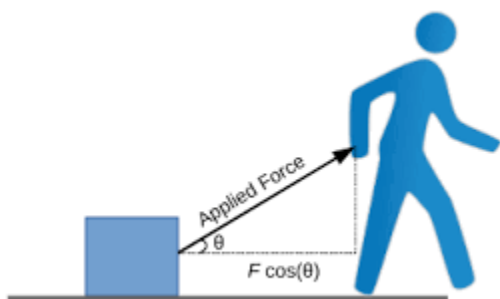
$$F = 100 \text{ N}, d = 8 \text{ m}$$

Since  $F$  and  $d$  are in the same direction,  $\theta = 0$ , [ $\theta$  is the angle of the force to the direction of movement], therefore

$$W = Fd \cos \theta$$

$$W = 100 \times 8 \times \cos 0$$

$$W = 800 \text{ J} \text{ [Since } \cos 0 = 1 \text{]}$$



When force acts in direction of motion of body, it is called **positive work** Direction of force and Direction of Motion in the same direction. When force acts opposite to direction of motion of body it is called **negative work** Direction of force and Direction of Motion are at angle of 180 degrees.

**Positive work:** Force and displacement are in the same direction, leading to motion.

**Negative work:** Force opposes the direction of displacement, slowing down or stopping motion.

**Zero work:** Force is applied, but there's no displacement, so no work is done.

### Net work

The net work done on an object in Physics is the sum of the work done by all the individual forces acting on that object.

If there are multiple forces acting on the object, the network ( $W_{\text{net}}$ ) is the sum of the individual works:

$$W_{\text{net}} = \sum_i W_i$$

### 6.1.2. Explanation of energy

Energy is a **scalar physical quantity**. Energy is generally defined as the potential to do work or produce heat.

Mechanical energy is the **energy that is possessed by an object due to its motion or due to its position**.

## 1. POTENTIAL ENERGY

Potential energy may be defined as the energy possessed by objects or bodies due to their position or state of strain or the position of their parts. Potential energy is energy deriving from position.

Potential Energy Formula is given by

$$P. E = m \times g \times h$$

Where  $m$  is the mass of the body,  $h$  is the height attained due to the body's displacement and  $g$  is the acceleration due to gravity, which is constant on earth. It is expressed in Joules.

### **Example:**

A ball of mass 2kg is kept on the hill of height 3km. Calculate the potential energy possessed by the ball.

### **Answer:**

Mass of the body ( $m$ ) = 2kg, Height ( $h$ ) = 3km,

Potential Energy possessed by the body =  $m \times g \times h$

Where  $g = 9.8 \text{ m/s}^2$

$$\therefore \text{Potential Energy} = (2\text{kg}) \times (9.8\text{m/s}^2) \times (3 \times 1000\text{m}) = 58800\text{J}.$$

## 2. Kinetic energy

**Kinetic energy** is the form of energy possessed by moving bodies. Kinetic energy of a body is dependent upon both the body's mass and speed. In mechanics, for a point particle, it is mathematically defined as the amount of work done to accelerate the particle from zero velocity to the given velocity;

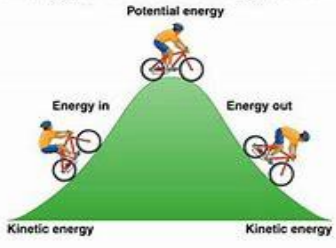
$$\text{Kinetic Energy } E_k = \frac{1}{2} mv^2$$

In Physics, mechanical work is the amount of energy transferred by a force acting through a distance. If a force  $F$  is applied to a particle that achieves a displacement  $d$ , the work done by the force is defined as the product of force and displacement:

$$W = F \cdot d$$

**Mechanical Energy**

- **total** potential energy & kinetic energy of an object
- $ME = PE + KE$



The **law of conservation of energy** states that **energy can neither be created nor be destroyed**. Although, it may be transformed from one form to another.

### Work-Energy Theorem

The **work-energy theorem** states that the work done by all forces acting on a particle equals the change in the particle's kinetic energy.

The work  $W$  done by the net force on a particle equals the change in the particle's kinetic energy

$$W = \Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Where  $v_f$  is the final velocity and  $v_i$  is the initial velocity

### Example:

A 145g baseball is thrown with a speed of 25m/s.

(a) What is its kinetic energy?

(b) How much work was done on the ball to make it reach this speed, if it started from rest?

### Answer:

a) The kinetic energy is  $EK = \frac{1}{2}mv^2 = 45J$

b) Since the initial kinetic energy was zero, the net work done is just equal to final kinetic energy, 45J.

### 6.1.3. Explanation of power

In Physics, power is the rate at which work is done or the rate at which energy is transferred or transformed. Power (P) is a scalar quantity and is defined as the amount of work (W) or energy (E) divided by the time (t) taken to perform the work or transfer the energy:

$$P = \frac{W}{t}$$

$$P = \frac{E}{t}$$

The unit of power in the International System of Units (SI) is the watt (**W**), and 1 watt is equal to 1 joule per second (1W=1J/s).

#### Examples

**Problem 1:** An electric machine makes use of 300 J of energy to do work in 10s. How much power does it use?

**Answer:**

Known: Work done = W = 300 J,

Time taken t = 10 s.

Power used by it is given by

$$P = \frac{W}{t} = \frac{300J}{10s} = 30Watts$$

**Problem 2:** John is who has a mass of 60 kg runs up to 12m high in 40 seconds. Compute his power.

**Answer:**

Known: m (mass) = 60 kg,

h (Height) = 12 m,

t (time taken) = 40 seconds.

Power is given by:

$$P = \frac{\text{work done}}{\text{time taken}}$$

$$P = mgh/t$$

$$= \frac{60 \times 9.8 \times 12}{40} = 588 \text{ Watts}$$

### Energy dissipation

No system is perfect. Whenever there is a change in a system, energy is transferred and some of that energy is dissipated.

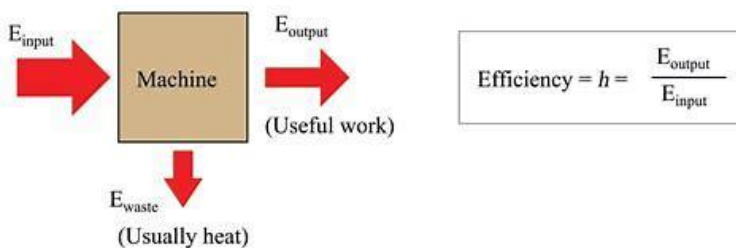
**Dissipation** is a term that is often used to describe ways in which energy is wasted. Any energy that is not transferred to useful energy stores is said to be wasted because it is lost to the surroundings.

**E.g.** 1. Electrical cables warming up

2. Friction which causes heating of the two surfaces.

### Efficiency of machines

**The efficiency of a machine** indicates how well its input energy is converted to useful output energy or work. It is a major factor in the usefulness of a machine and is the fraction or percentage of the output divided by the input. According to the Law of Conservation of Energy, the total output energy or work must equal the total input energy.



### Theoretical learning Activity

**In groups of four, discuss on the following problems**

1. A force of 20N pushing an object 5m in the direction of the force. How much work is done?
2. If you do 100 joules of work in one second (using 100 joules of energy). How much power is used?
3. A girl is carrying a bucket of water of mass 5kg. If she does 500J of work, to what height will she raise it?



## Points to Remember

- The net force is defined as is the sum of all the forces acting on an object.
- Kinetic energy is the form of energy possessed by moving bodies.
- Potential energy may be defined as the energy possessed by objects or bodies due to their position or state of strain or the position of their parts.
- The law of conservation of energy states that energy can neither be created nor be destroyed. Although, it may be transformed from one form to another.
- The work-energy theorem states that the work done by all forces acting on a particle equals the change in the particle's kinetic energy.
- Dissipation is a term that is often used to describe ways in which energy is wasted.



## 6.2 : Identification of types of energy

### 6.2.1. Energy

In Physics, energy is considered a **quantitative property**, which can be transferred from an object in order for it to perform work. Hence, we can define energy as the strength to do any kind of physical activity.

The energy source is a system, which produces energy in a certain way.

There are two kinds of energy sources;

#### 1. Primary sources.

#### 2. Secondary sources.

**Primary Sources** are from sources which can be used directly as they occur in the natural environment.

They include. 1. Flowing water 2. Nuclear 3. Sun 4. Wind 5. Geothermal (interior of the earth) 6. Fuels 7. Minerals 8. Biomass (living thing and their waste materials).

**Secondary sources** are energy sources that are generated from primary sources. For instance, electricity is a secondary source because it is generated for example from solar energy using solar panels or from flowing water using the turbines to generate hydroelectricity.

Other secondary sources of energy include; petroleum products, manufactured solid fuels, gases, heat and bio fuel.

## 6.2.2. Identification of sources of energy

### Renewable and non-renewable sources of energy

#### 1. Renewable energy sources

Examples include: **Biomass, Biogas, Geothermal, Wood waste, Hydropower, Wind, and Solar**

#### 2. Non-renewable energy sources

Examples are **coal, crude oil, natural gas, and uranium.**

### Explanation of two sources of energies:

#### 1. Renewable energy sources

A renewable energy source is an energy source which can't be depleted/exhausted. They exist infinitely i.e. never run out. They are renewed by natural processes.

However, some like trees they can also be depleted like trees and animals if used too much more than the natural process can renew them. So it's advisable to take precaution while using them, that is, they should be conserved.

#### 1. Non-renewable energy sources

These are sources which can be depleted because they exist in fixed quantities. So, they will run out one day.

Fossil fuels like coal, crude oil, natural gas are mainly made up of carbon. They are usually found in one location because they are made through the same process and material.

## 6.2.3. Formation of energy

#### 1. Renewable energy sources

(a) Wind energy

## Main parts of a modern wind turbine



Figure 44: A wind turbine

The term "**wind energy**" or "wind power" refers to the energy produced by wind. It is used to rotate turbines that convert kinetic energy into other forms that can be used to do specific tasks such as grinding grain, pumping water or generate electricity to power homes, businesses, schools, and industries.

### (b) Water energy



Figure 45: Nyabarongo dam in Mushishiro in Muhanga district.

Moving water mainly produces energy in the form of wave power, tidal barrage, and hydroelectric power.

### **Wave energy**

The water in the sea rises and falls because of waves on the surface. Wave machines use the kinetic energy in this movement to drive electricity generators.

### **Tidal barrage**

Huge amounts of water move in and out of river mouths each day because of the tides. A tidal barrage is a barrier built over a river estuary to make use of the kinetic energy in the moving water. The barrage contains electricity generators, which are driven by the water rushing through tubes in the barrage.

### **Hydroelectric power (HEP)**

Hydroelectric power stations use the kinetic energy in moving water. The water comes from vast reservoirs behind a dam built across a river valley. The water high up in the dam possess gravitational potential energy. This is transformed to kinetic energy as the water rushes down through tubes inside the dam. The moving water drives electrical generators which are built at the base of the dam.

### **(c) Solar energy**



*Figure 46: solar panel*

Solar power is energy from the sun. It is a powerful source of energy. Without it, there would be no life. It has been considered Earth's main source of energy for many years because of the vast amounts of energy that it makes freely available, if harnessed by modern technology.

#### (d) Geothermal energy

Geothermal energy come in the form of hot steam from underground. It is clean and sustainable. Resources of geothermal energy range from the shallow ground to hot water and hot rock found a few miles beneath the Earth's surface, and down even deeper to the extremely high temperatures of molten rock called magma. The steam is used to generate electricity.

### 2. Non-renewable energy.

Fossil fuels are mainly made up of carbon. It is believed that fossil fuels were formed over 300 million years ago when the earth was a lot different in its landscape. It had swampy forests and very shallow seas. This time is referred to as 'Carboniferous Period'.

The fossil fuels are **coal**, **oil** and **natural gas**. They are fuels because they release heat energy when they are burned. They have chemical energy stored within them. Fossil fuels are usually found in one location as their formation is from a similar process.

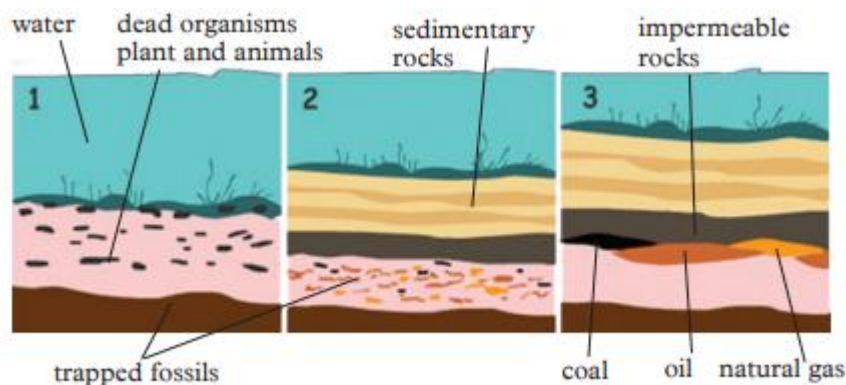


Figure 47: Formation of fossils fuels

Millions of years ago, Dead Sea organisms, plants and animals settled on the ocean floor and in the porous rocks. This organic matter had stored energy in them as they had used the solar energy to make foods through photosynthesis. With time, sand, sediments and

impermeable rock settled on the organic matter, trapping energy within the porous rocks. That formed pockets of coal, oil and natural gas. Movements in the earth and rock create spaces that force these energy types to collect in to well-defined areas. Using technology, engineers are able to drill down into the seabed to tap the stored energy, commonly known as crude oil.

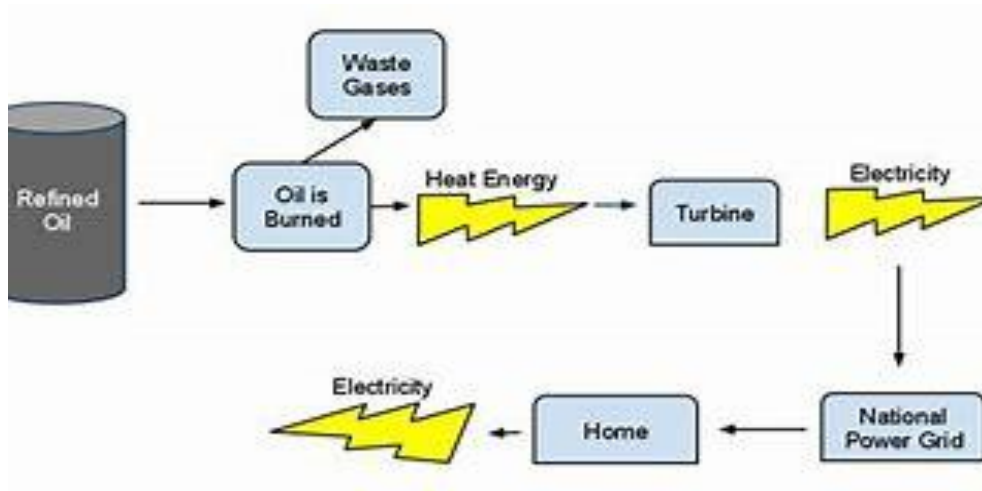


Figure 48: Energy transformation diagram



### Theoretical learning Activity

In groups of four, trainees discuss on the following questions:

#### 1. To identify and define energy sources

Material

- A chart showing pictures of different sources of energy Steps

1. Look at the pictures in Fig. 6.1.

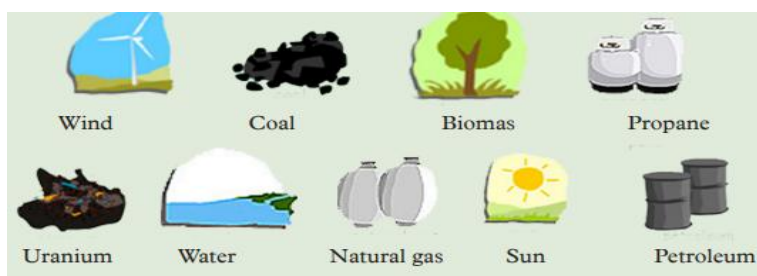


Fig 6.1

2. What name collectively describes the objects in the pictures?
3. Discuss with your classmates the meaning of the term's 'source' and 'energy source'.
4. With the help of your teacher, compile your findings and note them in your books.

## 2. To classify energy sources

### Steps

1. Distinguish between renewable and non-renewable sources.
2. Revisit activity 6.1 and categorize the energy sources shown in the picture as either renewable or non-renewable sources.

## 3. To find out what renewable resources are

### Steps

1. Discuss with your classmate the meaning of renewable sources of energy.
2. Identify at least three (3) characteristics of renewable sources of energy.
3. Describe three examples of renewable sources of energy in Rwanda and the World.



### Points to Remember

- Energy is considered a **quantitative property**, which can be transferred from an object in order for it to perform work.
- There are two kinds of energy sources;
  1. Primary sources.
  2. Secondary sources.
- Primary Sources are from sources which can be used directly as they occur in the natural environment.
- **Secondary sources** are energy sources that are generated from primary sources.
- Identification of sources of energy

#### **Renewable energy sources**

Examples include: **Biomass, Biogas, Geothermal, Wood waste, Hydropower, Wind, and Solar**

- **Non-renewable energy sources**

Examples are **coal, crude oil, natural gas, and uranium.**



## 6.3 : Analyzing relative advantages and disadvantages of various energy sources

### 6.3.1. Non-renewable energy

#### Advantages of using fossil fuels

Fossil fuels are relatively cheap and easy to obtain. This is the reason why most people prefer using gas or kerosene for cooking over electricity.

#### Disadvantages of using fossil fuels

- Fossil fuels are non-renewable energy resources. Their supply is limited and they will eventually run out.
- Fossil fuels release carbon dioxide when they burn, which adds to the greenhouse effect and increases global warming. Of the three fossil fuels, for a given amount of energy released, coal produces the most carbon dioxide and natural gas produces the least.
- Coal and oil release sulphur dioxide gas when they burn, which causes breathing problems for living creatures and contributes to acid rain.

### 6.3.2. Renewable energy sources

#### a. Wind energy

#### Advantages of wind energy

- Exploitation and utilisation of wind energy has no associated fuel costs.
- No harmful polluting gases are produced by wind energy.

#### Disadvantages of wind energy

- Wind farms are noisy and may cause noise pollution for people living near them.

- The amount of electricity generated depends on the strength of the wind. If there is no wind, there is no electricity.

### **b. Hydropower**

#### **Advantages of water energy**

- Water power in its various forms is a renewable energy resource and has no associated fuel costs.
- No harmful polluting gases are produced.
- Tidal barrages and hydroelectric power stations are very reliable and can be turned on quickly. **Disadvantages of water energy**

- It has been difficult to scale up the designs for wave machines to produce large amounts of electricity.
- Tidal barrages destroy the habitat of estuary species, including wading birds.
- Hydroelectricity dams may flood farmlands and push people away from their homes.
- The rotting vegetation underwater releases methane, which is a greenhouse gas that contributes to global warming and ozone layer depletion.

### **c. Solar energy**

#### **Advantages of solar energy**

- Solar energy is a renewable energy resource and it has no associated fuel costs.
- No harmful polluting gases are produced.

#### **Disadvantages of solar energy**

- Solar cells are expensive and inefficient, so the cost of their electricity is high.
- Solar panels may only produce very hot water in very sunny areas, and in cooler areas may need to be supplemented with conventional boilers.
- Although warm water can be produced even on cloudy days, neither solar cells nor solar panels work at night.

#### **d. Geothermal energy**

##### **Advantages of geothermal energy**

- **Environmentally Friendly.** Geothermal energy is more environmentally friendly than conventional fuel sources such as coal and other fossil fuels.
- **Renewable.** Geothermal energy is a source of renewable energy that will last until the Earth is destroyed by the sun in around 5 billion years.
- **Huge Potential.** Worldwide energy consumption is currently around 15 terawatts, which is far from the total potential energy available from geothermal sources.
- **Sustainable / Stable.** Geothermal provides a reliable source of energy as compared to other renewable resources such as wind and solar power.
- **Heating and Cooling.** Effective use of geothermal for electricity generation requires water temperatures of over 150°C to drive turbines.

##### **Disadvantages of geothermal energy**

- **Location Restricted.** The largest single disadvantage of geothermal energy is that it is location specific.
- **Environmental Side Effects.** Although geothermal energy does not typically release greenhouse gases, there are many of these gases stored under the Earth's surface which are released into the atmosphere.
- **Earthquakes.** Geothermal energy also runs the risk of triggering earthquakes. This is due to alterations in the Earth's structure because of digging.
- **High Costs.** Geothermal energy is an expensive resource to tap into, with price tags ranging from around \$2-\$7 million for a plant with a 1-megawatt capacity.



## Theoretical learning Activity

**In groups of four, trainees discuss on the following problems:**

1. What is renewable energy?
2. Why is renewable energy preferable?
3. Name:
  - (a) Two renewable sources of energy.
  - (b) Which component of sun's energy is responsible for drying clothes?
  - (c) Two forms of energy usually used at homes.
  - (d) The radiation emitted from a hot source.
  - (e) Two activities in our daily life in which solar energy is used.
  - (f) The kind of surface that absorbs maximum heat.
  - (g) The device that directly converts solar energy into electrical energy.
  - (h) The two elements which are used to fabricate solar cells.
4. What is the main cause of blowing of the wind?
5. State four characteristics of renewable sources of energy.
6. What are some ways that humans used renewable resources of energy centuries or even a millennia ago?
7. Research what kind(s) of renewable energy our country produces. Why is our country an optimal location for that form of renewable energy?
8. Choose a renewable energy resource. Brainstorm on five types of careers in that field.



## Practical learning Activity

### To make a simple wind turbine

#### Materials

- Manilla paper
- A pair of scissors
- Pencil
- A nail
- Stapler

#### Steps

1. Cut a square piece from the Manila paper.
2. Use a ruler to draw diagonal lines from corner to corner. Make a small mark along each diagonal line about 2 cm from the center of the square piece.
3. Cut along the diagonal lines toward the center until you reach the 2 cm mark.
4. Fold alternating corners onto the center and staple the layers together, but make sure to leave space between staples in the very center.
5. When all four 'blades' are folded in, push a straight nail through all the layers at the center. Remove the nail and push the pencil through the hole to act as the 'shaft'. The turbine is now complete (Fig 6.3). Make sure the turbine is free to rotate on the pencil

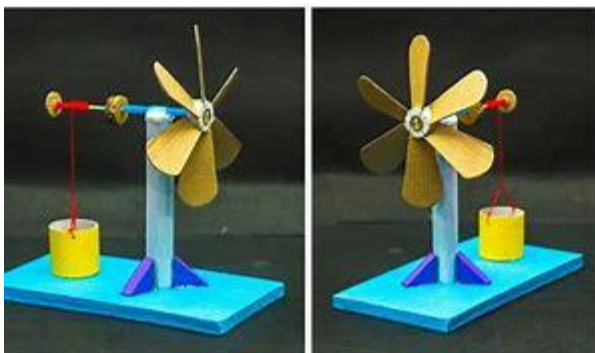


Fig. 6.3: A simple wind turbine

6. Hold the turbine in the direction of the wind. The wind currents blow the curved part of the blades, causing them to spin.



## Points to Remember

- **A renewable energy source** is an energy source which can't be depleted/exhausted. They exist infinitely i.e. never run out. They are renewed by natural processes.
- **Non-renewable energy source.** These are sources which can be depleted because they exist in fixed quantities. So, they will run out one day.
- The term "**wind energy**" or "wind power" refers to the energy produced by wind.
- **Geothermal energy** comes in the form of hot steam from underground.
- Hydroelectric power stations use the kinetic energy in moving water.



## Learning outcome 6 : Formative Assessment

### Written assessment

For questions 1 - 10, select the question that you think it is right.

1. Energy sources that once used can replenish themselves and can be used again and again are termed as

- |                  |            |
|------------------|------------|
| A. Non-renewable | C. Finite  |
| B. Renewable     | D. Kinetic |

2. Which of the energy sources listed is not a renewable source of energy

- |                      |          |                |
|----------------------|----------|----------------|
| <b><u>A. Oil</u></b> | C. Wind  | (e) Geothermal |
| B. Solar             | D. Tidal |                |

3. What is the other name for non-renewable

- |                  |                  |             |
|------------------|------------------|-------------|
| A. Non-renewable | <b>B. Finite</b> | C. Infinite |
|------------------|------------------|-------------|

4. Energy sources that once used cannot be replenished are called;

- |                         |              |             |
|-------------------------|--------------|-------------|
| <b>A. Non-renewable</b> | B. Renewable | C. Infinite |
|-------------------------|--------------|-------------|

D. Potential

E. Kinetic

5. What natural source is harnessed to generate hydro-electric power (HEP)?

A. Wind

C. Light

**B. Water**

D. Heat

6. What is the other name of the renewable energy source generated from using volcanic heat found under the earth's surface?

A. Wind

C. Tidal

B. Hydro-electric  
power (HEP)

**D. Solar**

E. Geothermal

7. What is the name of the renewable energy supply generated by capturing sunlight in panels that convert the sunlight into electricity?

A. Wind

C. Tidal **D. Solar**

B. Hydro-electric power

E. Geothermal

8. Which statement below is not an advantage of tidal energy?

A. Tidal barrages have the potential to generate a lot of energy

B. Tidal barrages can double as bridges

**C. Tidal barrages can help to prevent flooding**

D. Tidal energy is renewable and once in use can be used for generations

9. What type of energy source comes from radioactive minerals such as uranium and releases energy when the atoms of the radioactive minerals are split by nuclear fission?

A. Biomass

D. Hydro-electric  
power

B. Natural gas

**E. Nuclear**

C. Geothermal

10. What type of energy source is formed from fossilized plants and is found sandwiched between other types of rock in the earth?

A. Oil

C. Geothermal

E. Nuclear

**B. Coal**

D. Biomass

10. Describe the advantages and disadvantages of using fossil fuels to generate electricity.

### **Solution**

#### **Advantages of Using Fossil Fuels for Electricity Generation:**

- Abundant and Accessible: Fossil fuels, such as coal, oil, and natural gas, are abundant and widely accessible in many regions, ensuring a stable energy supply.
- Cost-Effective: Fossil fuel-based power plants are often cost-effective to build and operate, contributing to their widespread use.
- Reliable Power Generation: Fossil fuel power plants can provide a continuous and reliable power supply to meet electricity demand.

#### **Disadvantages of Using Fossil Fuels for Electricity Generation:**

- Environmental Impact: Burning fossil fuels releases greenhouse gases, contributing to climate change and air pollution.
- Resource Depletion: Fossil fuels are finite resources, and their extraction can lead to environmental degradation and resource depletion.
- Dependency on Imports: Some regions heavily depend on imported fossil fuels, making them vulnerable to supply disruptions and price fluctuations.
- Non-Renewable: Fossil fuels are non-renewable, and their consumption contributes to the depletion of finite resources.

11. Describe how fossil fuels are formed.

### **Solution**

- Decomposition of Organic Matter: Plants and microscopic organisms, such as algae and bacteria, die and accumulate in swampy environments or ocean beds. The organic material includes carbon-rich compounds.

- Formation of Peat: Over time, layers of organic material build up, and the lower layers undergo partial decay in the absence of oxygen. This leads to the formation of peat, an accumulation of waterlogged and partially decomposed plant material.
- Conversion to Coal, Oil, or Natural Gas: As more layers of sediment accumulate over the peat, the heat and pressure increase. This geological transformation, known as diagenesis, causes the peat to undergo further chemical changes.

12. Name three compounds that are formed from the chemical compounds in petroleum.

**Solution**

- ✓ Gasoline (Petrol):
- ✓ Diesel Fuel:
- ✓ Kerosene:

13. If fossil fuels are still forming, why are they considered to be a non-renewable resource?

**Solution**

Fossil fuels are considered non-renewable resources despite the ongoing formation of some organic material because their natural formation process is extremely slow, taking millions of years. The rate at which fossil fuels are extracted and consumed by human activities far exceeds the rate at which new fossil fuels are being formed.

14. Solar energy has provided almost all the sources of energy on the earth. Explain.

**Solution**

Solar energy is the primary source that powers various natural processes and human activities on Earth. Harnessing direct solar energy and the indirect renewable forms it creates contributes to sustainable and cleaner energy solutions.

15. Explain two advantages and two disadvantages of using solar energy.

**Solution**

**Advantages of Solar Energy:**

**Renewable and Sustainable:** Solar energy is a renewable resource, meaning it is virtually inexhaustible as long as the Sun continues to shine. Harnessing solar power contributes to sustainable energy solutions and reduces dependence on finite fossil fuels.

**Environmentally Friendly:** Solar power generation produces minimal environmental impact compared to conventional energy sources. It doesn't release greenhouse gases or air pollutants, contributing to mitigating climate change and improving air quality.

### **Disadvantages of Solar Energy:**

**Intermittent Energy Production:** Solar energy production is dependent on sunlight availability, making it intermittent. Weather conditions, time of day, and seasonal variations can affect the amount of energy generated.

**High Initial Costs:** The initial costs of installing solar panels and related equipment can be relatively high. While the costs have been decreasing, the upfront investment can be a barrier for some individuals or businesses.

16. Explain why geothermal energy is unlikely to become a major energy source?

### **Solution**

Geothermal energy is unlikely to become a major global energy source due to its limited geographical availability, high initial costs, potential resource depletion, environmental concerns, and challenges in transmitting electricity from remote locations.

17. Describe three ways solar energy can be used.

### **Solution**

**Electricity Generation:** Solar cells convert sunlight into electricity.

**Solar Heating and Cooling:** Solar thermal systems provide heat for water and space.

**Passive Solar Design:** Building design uses sunlight for natural heating and cooling.

18. Explain how the generation of electricity by hydroelectric, tidal, and wind sources are similar to each other.

### **Solution**

Hydroelectric, tidal, and wind energy generation share a common principle: they all harness the kinetic energy of moving water or air to produce electricity. In these methods:

- ✓ Movement of Water or Air: Hydroelectric power relies on flowing water, tidal energy utilizes the rise and fall of tides, and wind power captures the movement of air.
- ✓ Rotation of Turbines: In all three, the kinetic energy of moving water or air rotates turbines connected to generators.
- ✓ Electricity Generation: The rotating turbines generate electricity as they convert the kinetic energy into mechanical energy and then into electrical energy through generators.

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